

**Project Report  
DAR-4**

# **Ubiquitous MIMO Multifunction Digital Array Radar ... and the Role of Time-Energy Management in Radar**

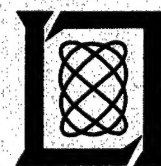
**D.J. Rabideau  
P.A. Parker**

**1 December 2003  
Issued 10 March 2004**

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**Lincoln Laboratory**  
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*LEXINGTON, MASSACHUSETTS*

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**Ubiquitous MIMO Multifunction Digital Array  
Radar ... and the Role of Time-Energy Management in Radar**

*D.J. Rabideau  
P.A. Parker  
Group 101*

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## ABSTRACT

Future navy surface radars will need large Power-Aperture-Gain (PAG) products so as to perform challenging Air and Missile Defense functions. Oftentimes, these radars will operate in littoral regions, where their large PAG products will cause strong clutter returns. Unfortunately, radar equipment specifications can become stressed by the need to detect small targets in such strong clutter. Stressing hardware specifications include dynamic range, phase noise, system stability, isolation and spurs. Moreover, the additional desire for Low Probability of Intercept (LPI) radar operation will also influence radar hardware design. Hence, as radar PAG increases, it may become increasingly difficult to design conventional radar equipment to operate as desired in littoral regions.

To partially address these issues, some future radar phased arrays (sometimes called “digital array radars”) will employ high degrees of aperture digitization. Typically, this digitization is performed near each of the receive elements in the array, enabling faster search rates, increased dynamic range, and improved adaptive beamforming performance. However, while such arrays offer many benefits, they still operate much like their predecessors, i.e., they look in *one* narrow sector at a time, and perform a *single* function at any given instant.

This report describes alternative approaches to operating phased array radars, especially digital arrays. These approaches involve transmit-array time-energy management; together, these alternative approaches are shown to ease the stressing hardware requirements described above. Time-energy managed digital arrays, for example, can be used to generate both highly focused transmit beams (e.g., for track) and broad transmit illumination (e.g., for search). Broad transmit illumination provides broad angular coverage, analogous to so-called “ubiquitous” radars (i.e., radars that “look everywhere, all the time”). This report describes how such broad transmit illumination can be provided by treating the transmitter and receiver subsystems of the radar as a single MIMO (Multi-Input, Multi-Output) system, or by employing more traditional approaches (e.g., beam spoiling or machine-gunning); pros and cons of each approach are discussed.

## **ACKNOWLEDGMENTS**

This report resulted from a nine-month effort funded by the Office of Naval Research (ONR). The authors would like to thank Drs. Bobby Junker and Mike Pollock, from ONR (Code-31), for sponsoring this work and our related research in the area of advanced multifunction RF digital array technology.

The authors also thank Dr. Harold Szu of ONR for sponsoring related work through ONR's 1999-2000 L-Band DAR program. That effort originally produced the idea of using the independent waveform generators in each DAR T/R module to generate orthogonal transmit waveforms, thereby enabling broad LPI search.

Finally, the authors would like to thank Drs. Phillip Phu and L. Cole Howard, of MIT Lincoln Laboratory, for interesting early discussions on the feasibility of this idea, as well as its benefits and its drawbacks.

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## 1. INTRODUCTION

To detect small RCS targets, military radars are usually designed for high peak power levels and large power-aperture-gain (PAG) products. Unfortunately, many of these same radars must operate in the presence of strong clutter, resulting in challenging requirements on system dynamic range, phase noise, stability, isolation, spurs, and other hardware-related specifications. Radar system design is further strained by requirements for fast search rates, multiple simultaneous functions (e.g., search and track), and high resolution in Doppler and angle. Moreover, the desire to be compatible with bi-static concepts, as well as the need to decrease harmful electromagnetic interference (EMI) and to reduce the probability of intercept by an enemy's Electronic Intercept (ELINT) system, further complicates the radar design.

To partially address these problems, radar arrays are being designed to use Digital Beam Forming (DBF) on receive [1,2]. In a DBF system, analog-to-digitization (A/D) conversion is performed near each of the receive elements in the array, thereby reducing A/D dynamic range requirements and facilitating the formation of multiple simultaneous receive beams (which enables faster search rates). While such arrays offer many benefits, they still operate much like earlier Analog Beam Forming (ABF) systems. Specifically, they look in *one* narrow sector at a time, and perform a *single* radar function (i.e., a specific type of search or track operation) at any given instant.

Many engineers have noted that if one were able to form truly *simultaneous* transmit beams in independent directions, then a flexible radar system could be built that would be capable of true simultaneous multi-function operation. By extension of the ideas presented in this report, we further note that by controlling the spatial region of illumination along with the power in each region, one could address most of the challenges listed above (i.e., dynamic range, phase noise, and so forth). Unfortunately, all attempts to build such a system (i.e., one capable of multiple simultaneous transmit beams) have suffered from problems associated with intermodulation distortion products (i.e., nonlinear distortion products that occur as the result of multiple radar signals being transmitted through the same device).

Instead of trying to solve the intermod problems described above, this report presents alternate techniques for utilizing phased arrays (especially digital arrays) that can be used to address the various hardware challenges described above. Principal among these techniques is a new radar operating concept-- one that takes advantage of the unique capabilities of *fully* digital arrays -- called MIMO DAR. (Note that a *fully* Digital Array Radar, or DAR, can perform digital beamforming on *both* transmit and receive; on transmit, waveforms are directly synthesized at each of the array elements). Previous works

have described how DAR systems can be used to synthesize a traditional “transmit beam.” However, by instead operating our DAR system in a Multi-Input, Multi-Output (MIMO) fashion, we show that the user can lower the peak power, while extending the integration time (so as to maintain system sensitivity). This “time-energy management” technique eases radar equipment specifications, and can offer improved performance. Moreover, since DAR systems can be operated in either the MIMO or the more conventional transmit modes (as needed), a great deal of flexibility is provided. The majority of this report is devoted to presenting and analyzing MIMO DAR.

This report also shows how many of the above mentioned hardware benefits can be attained through more traditional time-energy management techniques, such as beam spoiling or transmit beam machine gunning. However, to attain sizeable benefits these traditional techniques need to be used to a much larger degree than in the past (e.g., spoiling by factors much larger than the typical factor of 2 or 3 seen today). Moreover, the spatial energy distribution should also be controlled in certain ways to achieve the maximum desirable benefits.

Chapter 2 of this report describes digital arrays and time-energy management in detail. The concept of “ubiquitous” radar is reviewed, and the idea of a multifunction DAR containing a MIMO mode is described. Pros and cons of various time energy management techniques are also described.

Chapters 3 and 4 describe how time-energy management can ease radar equipment specifications and operating requirements. Chapter 5 describes how MIMO operation can improve angle resolution. Chapter 6 describes processing issues. Chapter 7 briefly describes issues relating to bistatic MIMO radar operation. A number of other considerations are briefly discussed in Chapter 8. Chapter 9 describes proof-of-concept experiments conducted at Lincoln Laboratory in 2003. Concluding remarks are given in Chapter 10.

## 2. DIGITAL ARRAYS AND TIME-ENERGY MANAGEMENT

Radar arrays are often classified based upon the type of beamforming hardware they use (i.e., analog vs digital). Analog beamforming (ABF) radars use analog components (e.g., phase-shifters, time-delay units, waveguides, and combiners) to form beams. As each ABF beam requires dedicated analog hardware, ABF radars typically produce very few beams. Digital beamforming radars, in contrast, use digital sampling and digital processors to form beams. In a DBF radar, forming multiple beams (on receive) is achieved by increasing the throughput of the digital processor, which is an easier task than adding analog components as required to form multiple beams with an ABF radar. Consequently, DBF is usually used when the system must form multiple beams.

Radar arrays are also classified based upon the types of beams used on transmit and receive – and more generally on how they manage their time and energy. At a most fundamental level, radars are usually designed to efficiently manage their scarce system resources, especially time and energy. To detect targets, radars must collect a sufficient amount of reflected target energy (i.e., Joules) so as to exceed an inherent receiver noise level. Radars do this by selecting a waveform and sending it through a set of signal amplifiers (each having a fixed maximum power level in Watts, i.e. Joules/sec.) and antennas (each having a fixed maximum gain). To ensure detection, then, radars must adjust the duration of their transmitted waveform so that (upon integration in the receiver) the required energy level is achieved for targets of interest.

In principal, the radar designer's choice of beamshape & waveform can be used to *spread energy over space and time* in many interesting and different ways, all whilst maintaining a constant sensitivity level – if desired. Transmit illumination may be (a) highly focused (e.g., using a focused beam), (b) slightly defocused (e.g., using traditional beam spoiling and machine gunning techniques), (c) completely defocused (e.g., using an omni transmit antenna as in “ubiquitous radar”), or (d) anywhere in between (e.g., using a Multifunction DAR with MIMO modes). When the transmit illumination is defocused, receive beams must be chosen to span the volume of space illuminated by the transmitter. Typically, this is done by using multiple, highly focused receive beams (collectively spanning the region of transmit illumination) as opposed to fewer defocused receive beams (the former requires more digital processing, but eliminates some losses). A number of these modalities are illustrated in Figure 1. The remainder of this chapter describes these modalities and their pros and cons.

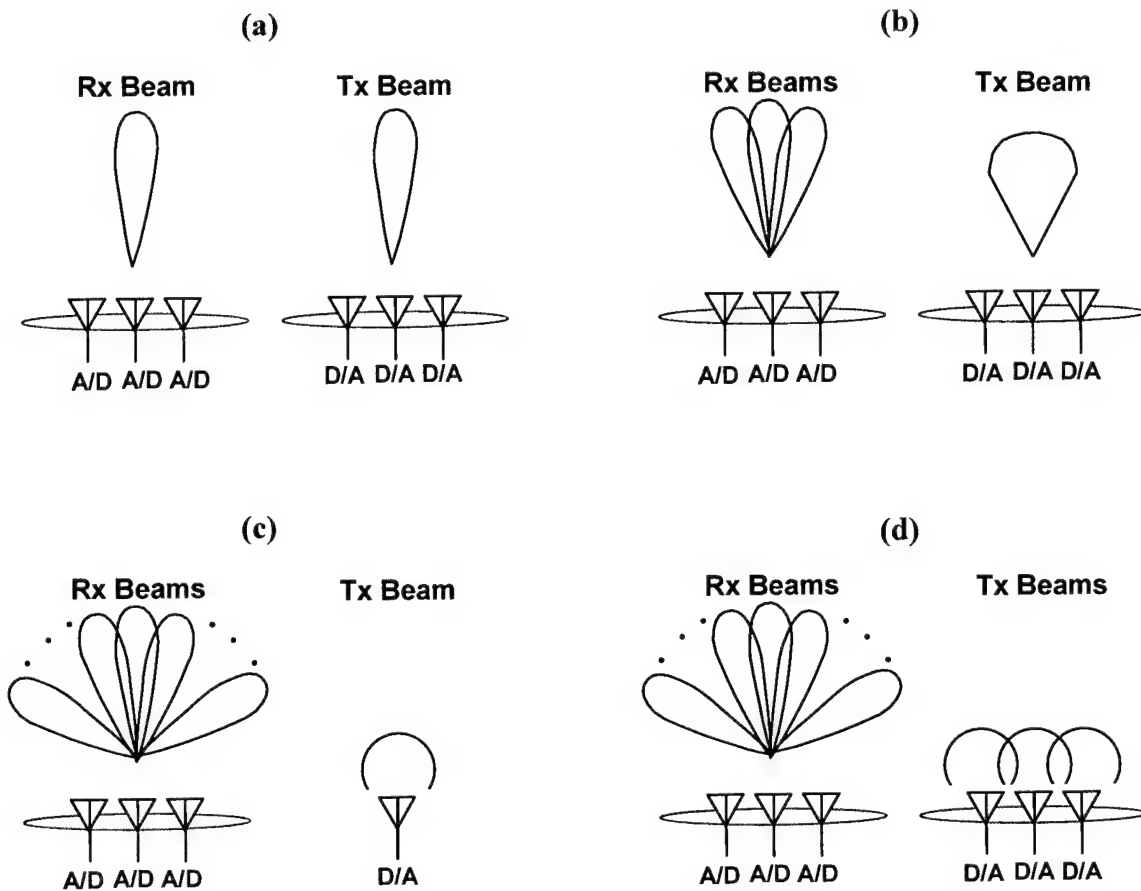


Figure 1. Potential DAR modes.

(a) Focused single-beam DBF, (b) Spoiled transmit beam, multiple DBF receive beams, (c) Ubiquitous, (d) MIMO.

## 2.1 ANALOG BEAMFORMING RADARS

Historically, most phased array radars have been designed to use large, high-gain antennas combined with short bursts of energy. High gain transmit antennas effectively focus the transmitted

energy into a small angular sector (e.g., one “beamwidth”). Thus, as long as targets of interest lie within this sector, the focusing of energy allows short transmission and integration times to be employed. *Short integration times, in turn, are sometimes very desirable for detection and estimation*<sup>1</sup>. Short integration times also allow the antennas to be rapidly re-steered to other angular sectors. This rapid re-steering of antennas can be used to increase the angular coverage of the system. Furthermore, rapid re-steering of antennas allows the radar to rapidly switch between multiple functions (e.g., searching the horizon and/or other areas, tracking targets, and target illumination).

Historically, most phased arrays have employed analog beamforming technology. In an analog beamforming radar, pulses are emitted using a highly focused transmit beamforming network comprised of waveguides and time-delay units (or analog phase shifters). Later, pulse returns (from targets and/or clutter sources) are collected by an equally focused receive beamforming network. After beamforming, a receiver at the output of the beamforming network is used to downconvert and sample the received signal.

This design philosophy (i.e., high-gain analog beamforming antennas with short-bursts of energy) is not without problems. First, observe that beamforming is done prior to the receiver; consequently, the receiver needs to support very high dynamic range levels, so as to accommodate strong interference sources that may lie within the mainlobe of the receive beam [2]. Second, while rapid re-steering of antennas increases angular coverage, functions are still performed sequentially, not simultaneously. Often, there are “not enough microseconds in a second” to sequentially perform all the desired functions over the desired angular sector. This problem is exacerbated by the need to maintain receiver isolation levels (which typically prevent you from turning-on receivers whilst transmitting<sup>2</sup>). Performing diverse functions can also be inefficient. A good example is the need to perform both (1) rapid revisit-rate close-in search as well as (2) long-range detection of small targets (at a slower rate). To detect small, long-range targets, a large aperture, high power transmitter could be used. However, such antennas will have a very narrow beamwidth, requiring many beams to cover a large angular sector. Since the minimum time-

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<sup>1</sup> As an obvious example of this, consider the detection of a target that is moving in an unpredictable fashion during the observation interval. This motion limits the length of time over which a target’s reflections can be coherently integrated at the receiver.

<sup>2</sup> This is analogous to the near-far problem CDMA systems.

per-beam is limited for practical reasons, this results in low revisit rates and long search times, even when performing functions that require less sensitivity (such as close-in search). See Figure 2.

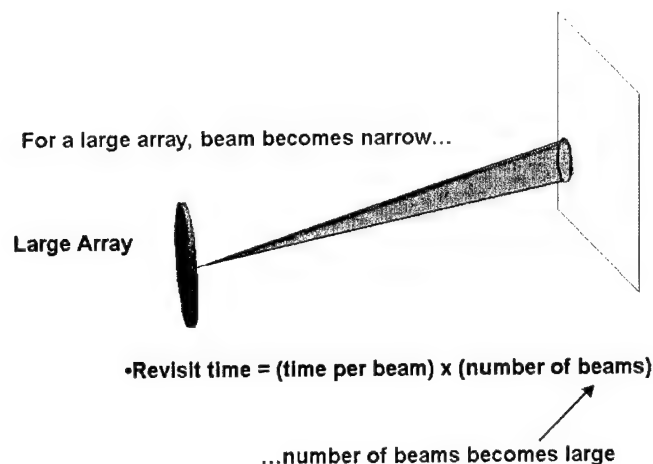


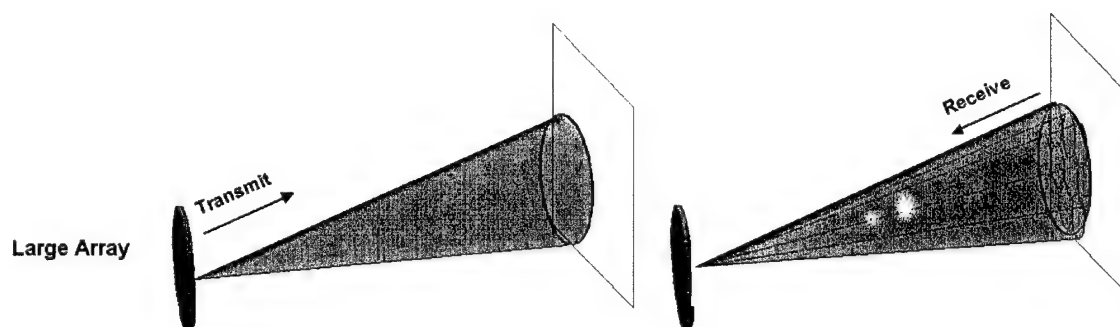
Figure 2. Searching a large angular sector using a single narrow beam will take a long time, resulting in low revisit rates.

## 2.2 DIGITAL BEAMFORMING RADARS AND DIGITAL ARRAY RADARS

To address these problems, DBF and DAR systems digitize the receive aperture (i.e., they use many digital receivers, one at each element or small subarray as shown in Figure 1a). With regard to receiver hardware specifications, aperture digitization reduces the gain on the interference entering each receiver. As a result, DBF systems reduce the receiver dynamic range requirement [1,2]. The various digital receiver outputs, of course, are then combined using digital beamforming (DBF) techniques.



With regard to Revisit Time (a.k.a. Frame Time), DAR systems can speed up their search rates by spoiling their transmit beam (thereby introducing an angular spread to the transmitted energy)<sup>3</sup>. This, combined with the formation of multiple simultaneous beams on receive, can be used to interrogate multiple beam positions simultaneously, thereby reducing the time needed to search a volume of space (see Figure 1b and Figure 3).



*Figure 3. Searching a large angular sector using a spoiled transmit beam and multiple simultaneous receive beams.*

As a very simple example of how the transmit beam can be spoiled, suppose we use one half of our total aperture to transmit a  $2\theta$  degree beamwidth, and use the full aperture to create two  $\theta$  degree receive beams [3]. By using 2 simultaneous receive beams, we effectively double the search rate. [With an active array, this approach is not efficient since the sensitivity will be decreased by a factor of four (due to loss in both transmit power and transmit aperture), while the search rate has increased by only a factor of two. However, more complex beam spoiling techniques exist and could be used to improve efficiency beyond the simplistic approach described above. Nonetheless, even these more complex beam-spoiling techniques are less than perfectly efficient, and can also suffer from undesirable ripple in the

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<sup>3</sup> ... This assumes the system has excess sensitivity, e.g., in the case of close-in search described in Section 2.1

mainlobe, high transmit sidelobes, and poor two-way sidelobes when large degrees of spoiling are used. In common usage, beamspoiling factors tend to be low (e.g., from 2:1 to 4:1).]

Note that for a 1-D aperture, spoiling by  $X:1$  implies the production of a beam having half-power beamwidth (HPBW) equal to  $\theta \cdot X$ , where  $\theta$  is the fully focused HPBW of the antenna. For a 2-D aperture, “ $X:1$  beamspoiling” refers to the process of increasing the antenna’s *half-power beam cross-sectional area* (HPBC) by a factor of  $X$ . Note that efficient  $X:1$  beamspoiling is achieved when the gain across the HPBC is reduced by a factor of  $X$  with respect to an unspoiled beam. As noted above, efficient spoiling can be difficult to achieve for various reasons.

Finally, note that it is possible to obtain most of the benefits of spoiled transmit beams without incurring the specific efficiency losses associated with beamspoiling. This is possible via a technique called “machine gunning.” Instead of defocusing the transmitted beam and transmitting a long pulse, a machine gunning radar transmits a sequence of short subpulses. The subpulses are transmitted rapidly (“machine gunned”); during the very short interval between subpulses the transmit beamformer is re-steered so that the aggregate set of subpulses spans a broader angular sector. To keep the overall duty cycle of the radar fixed, each subpulse in a machine gunning radar must be much shorter than the pulses of a conventional radar. With  $X$  subpulses, each subpulse is  $1/X$  times shorter. Furthermore, slight inefficiencies exist due to the time needed to re-steer the beamformer between subpulses. However, the biggest limitation is that as the sub-pulses get shorter, their time-bandwidth product decreases. Consequently, machine gunning systems may pay a price in terms of their range sidelobes, spectral containment, etc.

Of the radar systems (or prospective systems) that use transmit beamspoiling and/or machine gunning techniques today, all seem to employ relatively low degrees of spoiling (i.e.,  $X$  is small, perhaps on the order of 4 or so). Moreover, their spatial energy spreading patterns are not chosen to reduce hardware requirements. As we shall see later in this report, this is suboptimal from a hardware perspective. In fact, applying higher levels of spoiling along with optimizing the spatial spreading pattern, can, at least in theory, be used to address some of the hardware issues described in Chapter 1.

## 2.3 UBIQUITOUS RADAR

DBF systems, as described above, are not without problems. For one thing, radar functions are still performed sequentially, not simultaneously. Consequently, there are often “not enough microseconds in a second” to execute all desired radar functions. Secondly, the time-energy management techniques described above (i.e., beamspoiling and machine gunning) are typically not applied in ways that address a number a significant hardware and system challenges.

As an alternative to the traditional DBF architectures, the “ubiquitous radar” architecture depicted in Figure 1c has been proposed [4]. This architecture addresses some of the problems described above by using a low-gain transmit antenna (i.e., one having a broad transmit beam), combined with highly directive contiguous receive beams. Since the transmitter illuminates a very broad angular sector, the radar is able to see “everywhere all the time.” Due to the lower transmit gain, however, longer integration times will be needed to maintain system sensitivity. While sometimes undesirable, long integration times do have some benefits, such as increased Doppler resolution (which, among other benefits, can improve clutter cancellation). Furthermore, the broad angular coverage means that antennas do not need to be re-steered to cover a large angular sector.

In the limit, such a system would employ an omnidirectional antenna on transmit, along with a large number of contiguous (non-scanning) beams on receive, see Figure 4. While such systems were first considered in the 1960’s, the analog beamforming technology of the time made the design of the receive beamformer quite complex. Today, interest has been renewed [4] due to the modern possibility of digital beamforming at the receiver.

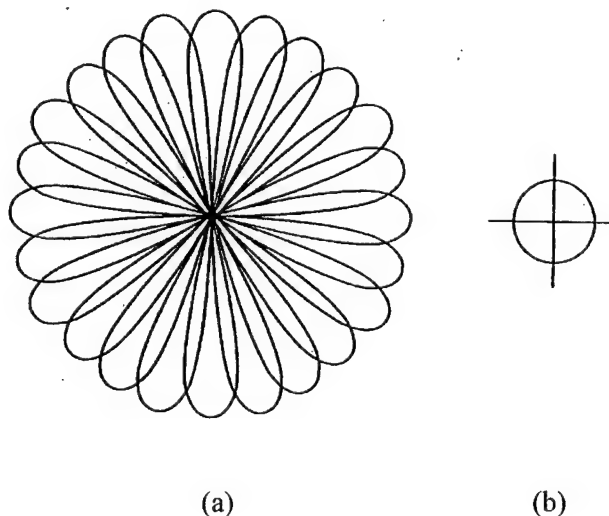


Figure 4. Ubiquitous Radar. (a) Multi-beam receive antenna, (b) Omni transmit pattern.

Unfortunately, this design philosophy (i.e., using a single low-gain transmit antenna, long waveforms, and multiple narrow receive beams) has its own problems. Notably, (1) the required

integration times can sometimes be extremely long and poorly suited to some important radar functions (e.g., in time-critical, high precision tracking modes), and (2) with an active transmit aperture, the system will suffer from both lower gain and lower power (which further increases the required integration times).

## 2.4 MIMO DIGITAL ARRAYS AND HIGH LEVELS OF TIME-ENERGY MANAGEMENT

Fortunately, some desirable attributes of the “ubiquitous radar” architecture can also be achieved by operating a traditional phased array transmitter in a time-energy managed mode that is tailored to produce a very high degree of spatial spreading. One way to do this would be to employ beamspoiling or machine gunning techniques, with  $X$  chosen to be much larger than the factor of 2 to 4 that is common today. More will be said on this in subsequent chapters. However, as noted earlier, we must not forget that there are certain reasons why these techniques are usually not employed with such high degrees of spoiling.

Fortunately, we argue that DAR systems can also achieve high degrees of spatial energy spreading by exploiting their unique transmit hardware (this topic will be the focus of much of this report). Specifically, a fully digital array can usually synthesize an independent waveform at each transmit module. There are many ways to design a DAR module so that this is possible; Figure 5 below illustrates one such approach (note that this is the high-level architecture proposed during the ONR-sponsored MIT/LL L-band DAR program several years ago [5]). Independent transmit waveforms are possible because each T/R module contains its own DDS-based waveform generator.

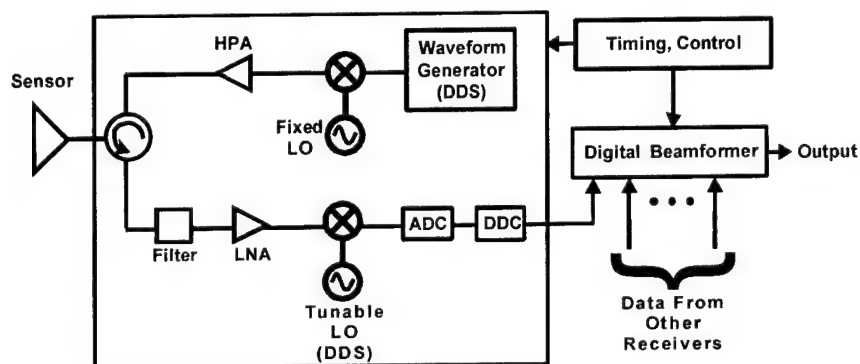


Figure 5. One possible architecture for a DAR Transmit/Receive Module.

To achieve high levels of spatial energy spreading, our approach utilizes the independent transmitters and digital receivers of the DAR as a Multi-Input, Multi-Output (MIMO) system, as depicted in Figure 1d. On transmit, the DAR aperture operates as  $X$  low-gain elemental transmitters, each radiating a unique, orthogonally coded waveform. The  $x^{\text{th}}$  transmitter radiates waveform  $w_x(t)$ . Note that due to the orthogonality of each waveform, the various emitted signals will not combine coherently in space to form a single focused beam; instead, the radiated energy will effectively cover a broad angular sector (as determined by the elemental radiator's spatial pattern). Assuming isotropic elemental radiators, the aggregate waveform incident upon any target can be represented as

$$w_{TGT}(t) = \alpha_1 \sum_{x=1 \text{ to } X} w_x(t - \tau_x)$$

where  $\tau_x$  is the time for the  $x^{\text{th}}$  waveform to propagate from the  $x^{\text{th}}$  transmitter to the target's position, and  $\alpha_1$  is an overall attenuation factor (assumed to be the same for all transmitters, for sake of simplicity). For ease of exposition, let us make the usual narrowband assumption (this is not a requirement, but simplifies the discussion). The aggregate waveform is then:

$$w_{TGT}(t) = \alpha_2 \sum_{x=1 \text{ to } X} w_x(t) a_x(\phi, \theta)$$

where the target angle is  $(\phi, \theta)$ ,  $\alpha_2$  is a complex scalar, and  $a_x(\phi, \theta)$  represents the  $x^{\text{th}}$  element of the usual  $X \times 1$  "transmit array response vector." This waveform then propagates back to the DAR, where it is received. The received waveform is then:

$$\mathbf{x}(t) = \alpha \mathbf{b}(\phi, \theta) \sum_{x=1 \text{ to } X} w_x(t) a_x(\phi, \theta) + \mathbf{e}(t) \quad (1)$$

where  $\alpha$  is a complex scalar,  $\mathbf{b}(\phi, \theta)$  is the usual  $N \times 1$  "receive array response vector," and  $\mathbf{e}(t)$  is a  $N \times 1$  vector of noise at time  $t$ . [Note that for simplicity, (1) has implicitly assumed a monostatic or pseudo monostatic aperture configuration; this is not strictly required].

On receive, the signal at each individual DAR receiver is processed through a bank of  $X$  matched filters, (e.g., see Figure 10 in Chapter 3.3). Each filter is matched to one of the transmitted waveforms, thereby isolating the returns due to a single transmit signal. This produces a total of  $X \cdot N$  matched filter outputs. Stacking the results into a vector, the signal is expressed as

$$\mathbf{x}(t) = \alpha (\mathbf{b}(\phi, \theta) \otimes \mathbf{a}(\phi, \theta)) \delta(t) + \mathbf{n}(t)$$

where  $\otimes$  is the Kronecker product and  $\mathbf{n}(t)$  is a  $(X \cdot N) \times 1$  noise vector. Since the locations of each transmit and receive element are known, these  $X \cdot N$  signals can be phased and combined (analogous to normal transmit and receive beamforming) to form beams in one or more directions<sup>4</sup>. For example, to form an unweighted beam in direction  $(\phi, \theta)$ , we multiply

$$y(t) = (\mathbf{b}(\phi, \theta) \otimes \mathbf{a}(\phi, \theta))^H \mathbf{x}(t) \quad (2)$$

More will be said about implementing (2) in Chapter 6. Note that further integration (i.e., Doppler processing) is used to maintain sensitivity, as desired<sup>5</sup>.

This basic MIMO idea is illustrated via Matlab simulation in Figure 6 below. This figure shows the recovered beampatterns formed with a 50 element uniform linear array. In the simulation, each transmit element used an orthogonally coded waveform, and 50 receive beams were formed simultaneously (with a relatively long integration time used to maintain sensitivity comparable to that of sweeping a single beam at a time using the full PAG of the radar).

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<sup>4</sup> Colleagues at Lincoln Laboratory are developing a related technique that eliminates the need for knowledge of element positions, relying instead on estimation. This is desirable when the transmitter and receive have unknown relative positions. Their work will appear in a future paper.

<sup>5</sup> Since the transmitted waveforms do not combine coherently in space, there is a loss in sensitivity equal to the loss in transmit array gain. This reduced sensitivity can be restored by integrating longer. Note that longer integration times do not necessarily imply longer Revisit Times during radar search modes. This is because defocusing the transmit illumination allows many beam positions to be searched simultaneously.

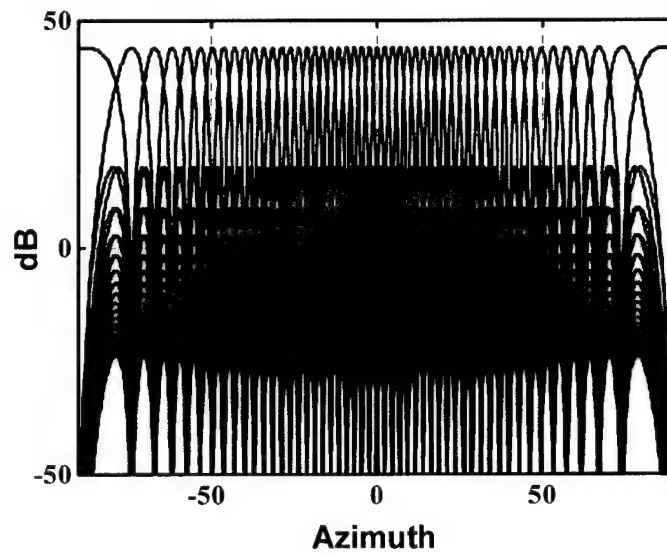
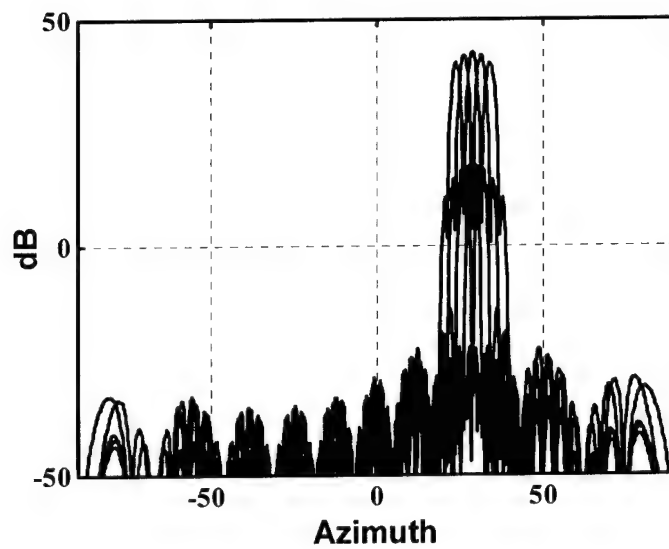


Figure 6. Effective two-way (transmit/receive) patterns, illustrating the ability to provide ubiquitous coverage.

Figure 7 illustrates how the system can be used in a sectored fashion. Here, the same 50 element array was partitioned into large subarrays. Each subaperture constitutes a medium-gain, medium directivity transmitter. Each subarray was used to emit a coded waveform (within each subarray, the waveform was phase-steered to a desired direction chosen to be 38 degrees from broadside). On receive, the waveforms are combined to form narrow beams spanning the subarray mainlobe with a relatively short integration time needed to maintain sensitivity.



*Figure 7. Effective two-way (transmit/receive) patterns, illustrating the ability to operate in a backward compatible fashion.*

## 2.5 SUMMARY REMARKS

Table 1 summarizes several methods that can be used to spread the transmit energy spatially, thereby facilitating time-energy management. In the table, a large spreading factor,  $X$ , is assumed.



**TABLE 1**  
**Techniques for time-energy management by spreading the transmit illumination**  
**(assuming large  $X$ )**

METHOD	COMMENTS
Multiple independent, simultaneous beams on transmit (each using the full array)	<ul style="list-style-type: none"> <li>• Offers greatest flexibility; multiple simultaneous (diverse) functions.</li> <li>• Potential difficulties associated with intermods (which can be partially addressed through linear HPAs).</li> <li>• Potential losses if using linear HPA modes to address intermods.</li> <li>• Poor 2-way patterns (which can be addressed by using orthogonal waveforms for each beam).</li> </ul>
Transmit Beamspooling	<ul style="list-style-type: none"> <li>• Supports search, track-while-scan; can resort to focused transmit beam when needed.</li> <li>• Potential for transmit inefficiency associated with transmit taper (difficult to spread HPBC by <math>X:1</math> with <math>1/X</math> gain across mainlobe).</li> <li>• Problems controlling mainlobe and sidelobes.</li> <li>• Poor 2-way patterns.</li> </ul>
Transmit Machine Gunning	<ul style="list-style-type: none"> <li>• Supports multiple diverse functions.</li> <li>• Scaling to large <math>X</math> is difficult (due to small waveform time-bandwidth products, high range sidelobes).</li> <li>• Potential for poor 2-way patterns (addressed by employing orthogonal waveforms at each subpulse).</li> <li>• Switching time between subpulses results in performance loss.</li> </ul>
Omni transmit (i.e., ubiquitous radar)	<ul style="list-style-type: none"> <li>• Supports multiple diverse functions.</li> <li>• If using a dedicated omni transmit antenna, performs poorly in modes requiring short integration time.</li> <li>• If using subaperture of active transmit array, then lost efficiency (due to loss of both power and aperture).</li> </ul>
DAR with MIMO mode	<ul style="list-style-type: none"> <li>• Supports simultaneous search and track (if ubiquitous), or search and track-while-scan (if transmit subarrays are used); can resort to focused transmit beam when needed.</li> <li>• Requires independent waveform generators on transmit.</li> <li>• Good 2-way patterns, enhanced angle resolution.</li> <li>• LPI modes.</li> <li>• Other benefits (see following chapters).</li> </ul>

### 3. EASING EQUIPMENT SPECIFICATIONS

As mentioned in Chapter 1, radar equipment specifications are often determined by the need to detect small targets in the presence of large interference signals. Such equipment specifications include receiver dynamic range and isolation, LO phase noise and system stability, and transmitter spurs. In this chapter, we describe how various time-energy management techniques can be used to ease tough radar equipment requirements. As we shall see, many of these requirements depend on the peak power level of the radar and the illumination pattern of the transmitter. After introducing the various requirements, then, we shall describe how time-energy management techniques can be used to reduce clutter power (both at the receiver, and after the beamformer). Then, we relate this reduction back to various radar equipment parameters.

#### 3.1 CHALLENGING RADAR EQUIPMENT SPECIFICATIONS

Future military radars will need to operate in the presence of very large interference signals, preferably without loss of sensitivity. Operational Requirements Documents often state that these radars must be able to detect small RCS targets amidst strong background interference consisting of clutter, jamming, and/or EMI. Naval air defense radars, for example, will require large power-aperture-gain (PAG) products so as to ensure sufficient SNR for detection of small RCS objects (e.g., cruise missiles, or very long range ballistic missile components). However, at the same time these radars must operate in littoral regions (i.e., near coastlines) to support land operations. In such regions, the radar's large PAG product can result in very strong backscattered terrain clutter.

Desired performance levels are often difficult to achieve with current equipment. Radar receivers, for example, must be designed to amplify, mix, and sample very strong interference signals without introducing minute levels of distortion that might obscure weaker target signals. Hence, the receivers must have a large *dynamic range (DR)*.

Likewise, the transmitted and received signals must maintain sufficient phase stability as to allow cancellation of clutter via MTI techniques. Hence, the receivers and excitors must support a very high *clutter improvement factor (CIF)*.

In CW systems, moreover, the receivers must be *isolated* from the transmitters so as to avoid saturating on the transmitted signal and/or nearby clutter.

Finally, to comply with strict spectral emissions requirements, transmitter spurs<sup>6</sup> must not be allowed to combine coherently in space.

Fortunately, some measures can be taken to ease stressing equipment requirements while maintaining overall system performance. It is well known, for example, that digital receive arrays can be used to ease equipment requirements relating to receiver dynamic range [1]. In an analog beamforming radar system, the analog beamforming network can impart a large gain onto undesired interference signals. As a result, the radar receiver, which is connected to the output of the beamforming network, must be specified to accommodate this gain in dynamic range. Digital receive arrays, in contrast, use many digital receivers with each receiver sampling the output of an individual antenna element (or small subarray). This avoids the need for increased dynamic range that results from beamforming prior to the receiver. Instead, this gain is later imparted digitally, using digital beamforming.

A lesser-known fact is that digitizing the aperture on transmit can also ease equipment requirements. Moreover, we shall see that the benefits go beyond dynamic range (some of these benefits can also be achieved via simple transmit beamspoiling).

The remainder of this section describes how beamspoiling and/or MIMO digitization (on transmit) can be used to ease equipment requirements. We begin by describing how these techniques can be used to reduce clutter power levels, and then we describe how this reduction eases equipment specifications relating to dynamic range, isolation, and transmitter spurs. In a later section, we address issues relating to clutter cancellation.

### **3.2 DISTRIBUTED CLUTTER POWER**

Consider the idealized radar system characterized by the parameters in Table 2 (note that a number of potential loss factors have been omitted for sake of simplicity).

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<sup>6</sup> ... and intermods, in the case of simultaneous multifunction transmit systems

**TABLE 2**  
**Radar parameters**

<b>SYMBOLS</b>	<b>DEFINITION</b>
$P_T$	Full transmit antenna's peak power
$G_T$	Full transmit antenna's gain
$\theta_A$	Full transmit antenna's azimuthal beamwidth
$\theta_E$	Full transmit antenna's elevation beamwidth
$G_R$	Full receive antenna gain
$G_E$	Individual receive element (or subarray) gain
$G_M$	MIMO processing gain
$X$	Number of MIMO transmitters
$\lambda$	Wavelength
$k$	Boltzmann's constant
$T_0$	Noise temperature
$F$	Receiver noise figure
$B$	Receiver bandwidth
$B_S$	Bandwidth of a single transmitted pulse
$\tau$	Duration of a single transmitted pulse
$\sigma_0$	Clutter RCS
$\sigma_t$	Target RCS
$R_t$	Target range
$\tau_t$	Aggregate time corresponding to all target returns in dwell
$D_t$	Radar target detection threshold

### 3.2.1 Clutter Levels in Conventional ABF Radar

For starters, assume our radar is designed to use conventional analog beam forming (ABF) on both transmit and receive. As such, our radar generates a single focused beam on transmit, and then uses analog beamforming to form a receive beam prior to downconversion and digitization. At range  $R$ , the radar's transmit beam illuminates an area of approximate size  $\theta_A \cdot R$  by  $c\tau/2$ , where  $c$  is the speed of propagation (see Figure 8). Hence, the clutter power at the input to the receiver ( $CP$ ), integrated over the area of land illuminated by the pulse is:

$$\begin{aligned}
 CP_{ABF} &= \underbrace{P_T G_T \left[ \frac{1}{4\pi R^2} \right]}_{\text{Effective Tx power per unit area at the range of the land clutter}} \underbrace{\sigma_0 \frac{\theta_A R c \tau}{2}}_{\text{Effective size of uncompressed land clutter patch}} \underbrace{\left[ \frac{1}{4\pi R^2} \right] \frac{G_R \lambda^2}{4\pi}}_{\text{Amount of reflected signal captured at the receiver}} \\
 &= \frac{P_T G_T \sigma_0 \theta_A c \tau G_R \lambda^2}{2(4\pi R)^3}
 \end{aligned} \tag{3}$$

Note that the clutter power will be largest when the clutter source lies within the mainlobe of both the transmit and receive beams (i.e., when  $G_T$  and  $G_R$  are at their maximum values). For Naval surface radars, this occurs whilst the radar is performing search and track near the horizon, i.e., in a small number of "low" elevation beams.

Of course, the noise power at the receiver is given by  $kT_0FB$ . Consequently, the Clutter-to-Noise-Ratio (CNR) is defined as:  $[ \text{equation (3)} ] / kT_0FB$ .

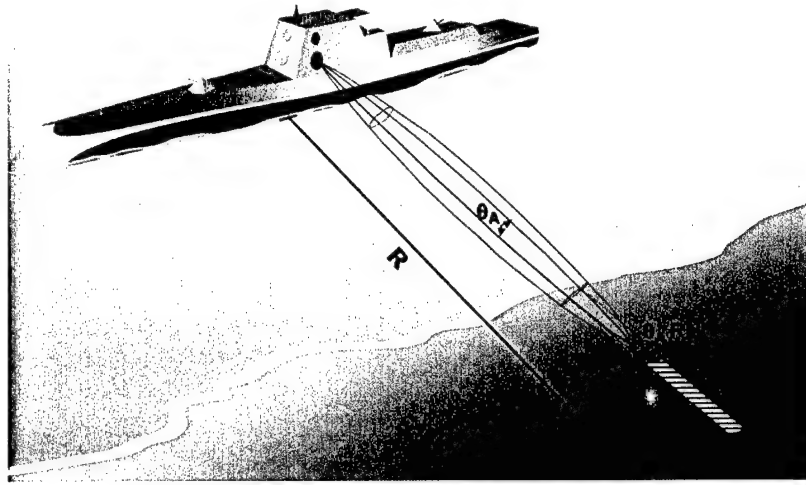


Figure 8. Clutter patch due to conventional transmit beamforming.

### 3.2.2 Clutter Levels in DBF Radar

Next, suppose our radar is redesigned to use digital beam forming (DBF) on receive. In a DBF radar, the receive array employs  $N$  digital receivers, with each receiver sampling the output of a single element (or small subarray of elements) in the array. This reduces the antenna gain prior to the receiver from  $G_R$  to  $G_E$ , a reduction of up to  $N$  (i.e.,  $G_E \geq G_R/N$ ). Hence, the clutter power at the receiver may be reduced by up to  $N$  as compared with (3), i.e.,

$$CP_{DBF} = \frac{P_T G_T \sigma_0 \theta_A c \tau G_E \lambda^2}{2(4\pi R)^3} \geq \frac{CP_{ABF}}{N} \quad (4)$$

Moreover, this reduction in clutter power is accompanied by an equivalent reduction in CNR at the receiver. Digital receiver outputs are then combined via digital beamforming techniques. Consequently, the clutter power at the output of this beamformer is the same as in (3).

### 3.2.3 Clutter Levels Due to Spoiled Transmit Beam

Next, suppose our radar is further redesigned to transmit a spoiled beam, and use multiple digital beams on receive (as described in Chapter 2). The transmit beam may be spoiled in azimuth, elevation, or both (see Figure 9). Assuming efficient  $X:1$  beamspoiling, the effect of beamspoiling on clutter power is as follows. First, the transmit gain is reduced from  $G_T$  to  $G_T/X$ . Likewise, the HPBC is increased by a factor of  $X$ . This implies the azimuthal beamwidth will be  $e\theta_A$  where  $e$  varies between 1 and  $X$ , depending on whether the beam is spoiled in elevation, azimuth, or both. Hence, the clutter power at the receiver will be:

$$CP_{BS} = \frac{P_T (G_T/X) \sigma_0 e \theta_A c \tau G_R \lambda^2}{2(4\pi R)^3} \geq CP_{ABF} \cdot \left( \frac{e}{XN} \right). \quad (5)$$

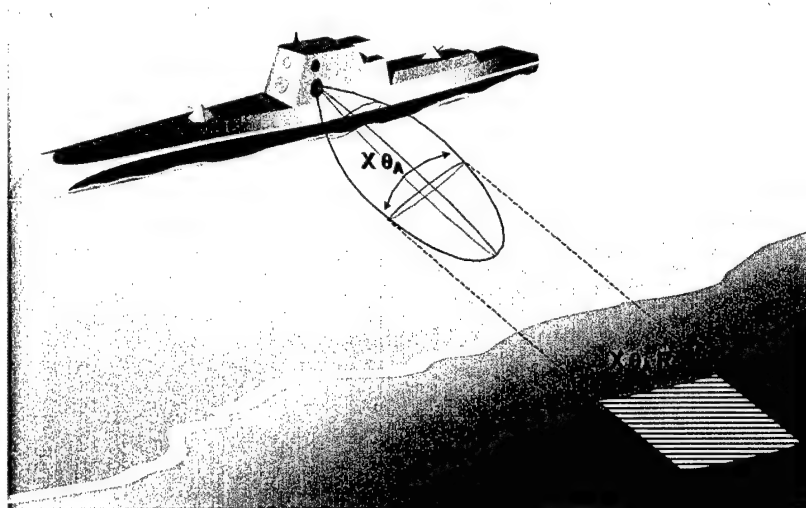
Likewise, after beamforming the clutter power will be,

$$\frac{P_T (G_T/X) \sigma_0 e \theta_A c \tau G_R \lambda^2}{2(4\pi R)^3} = CP_{ABF} \cdot \left( \frac{e}{X} \right) \quad (6)$$

where both (5) and (6) are referenced to a receiver noise level of  $kT_0FB$ . Furthermore, note that the clutter power will be minimized when the transmit beam is spoiled in elevation, corresponding to  $e = 1$  and a reduction in clutter power by a factor of  $X$ .

[It is interesting to note that spoiling *in elevation*, as suggested by (6), is exactly the opposite of the current Navy S-band DAR concept, which employs azimuthal beam spoiling near the horizon. Clearly, spoiling in elevation will have at least one disadvantage (i.e., as compared with azimuthal spoiling). Specifically, spoiling in elevation will increase the time required to sweep beams across the horizon. However, the added time is often small and an increase is probably acceptable. Note that the *overall* volume search frame time is approximately constant – assuming one desires a single (fixed) revisit rate throughout the entire search volume.]

(a)



(b)

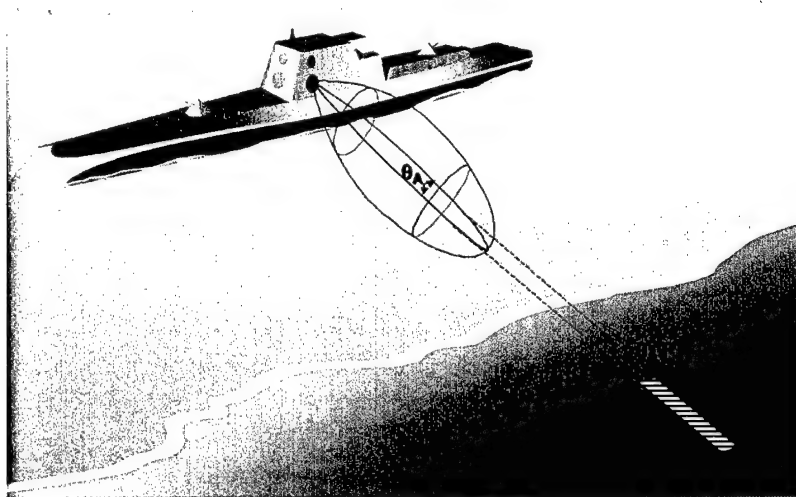


Figure 9. Clutter patch due to spoiled beam on transmit. (a) Azimuthal spoiling, (b) Elevation spoiling.



### 3.2.4 Clutter Levels Due to a Single MIMO Transmitter & Coherent Combinations of Multiple MIMO Transmitters

Finally, suppose our radar is designed to use MIMO on transmit along with DBF on receive. To analyze the impact of MIMO transmission on clutter power, let us assume the transmit array is partitioned into  $X$  independent sub-apertures, each transmitting an orthogonal waveform. The transmitted power per MIMO channel is then  $P_T/X$ . Likewise, the transmit gain per channel is  $G_T/X$ . Finally, the azimuthal beamwidth of each transmit channel will be  $e\theta_A$  where  $e$  varies from 1 to  $X$  depending on the aperture partitioning.<sup>7</sup>

According to the radar range equation, with  $P_T$ ,  $G_T$ , and  $\theta_A$  modified as in the previous paragraph, the clutter power at the receiver due to a *single* MIMO transmit channel will be:

$$CP_{M1} = \frac{(P_T/X)(G_T/X)\sigma_0 e\theta_A c\tau G_R \lambda^2}{2(4\pi R)^3} \geq CP_{ABF} \cdot \left( \frac{e}{X^2 N} \right) \quad (7)$$

where, as stated above,  $e$  varies from 1 to  $X$ . On receive, each transmitted signal may be spatially combined (exactly as in traditional digital beamforming). In doing so, the clutter power due to a single MIMO transmit channel will be:

$$\frac{(P_T/X)(G_T/X)\sigma_0 e\theta_A c\tau G_R \lambda^2}{2(4\pi R)^3} \quad (8)$$

where both (7) and (8) are referenced to a receiver noise level of  $kT_0 FB$ .

---

<sup>7</sup> Consider, for example, the partitioning of the transmit aperture such that all MIMO transmitters lie along the horizontal axis of the array face. In this case, the azimuthal beamwidth increases to  $X\theta_A$ , i.e.,  $e = X$ . In contrast, if the MIMO transmitters are co-aligned with the vertical axis of the array, the azimuthal beamwidth is  $\theta_A$ , i.e.,  $e = 1$ . Moreover, if the MIMO transmitters are dispersed evenly throughout the 2-D array aperture, the azimuthal beamwidth will be about  $\sqrt{X}\theta_A$ , or  $e = \sqrt{X}$ .

By assumption, there are a total of  $X$  orthogonal MIMO transmitters. Each of these transmitters will produce a clutter signal whose power (after digital beamforming) is given by (8). Due to the orthogonality of the transmitted waveforms, each of the DBF outputs can (in principal) be processed by a bank of matched filters (i.e., one filter matched to each transmit waveform). This filtering effectively separates the received signal (after DBF) into contributions due to the each of the various MIMO transmitters. These contributions may then be re-summed coherently for some direction(s) of interest.

By summing the matched filtering outputs corresponding to the  $X$  MIMO transmit channels, we provide a voltage gain of up to  $X$  on the clutter (i.e., a power gain of up to  $X^2$ ), and we effectively focus a transmit beam. However, combining MIMO channels can also increase the noise power by a factor of  $X$ . Hence, after both traditional digital beamforming and MIMO combining, the clutter power (referenced to  $kT_0FB$ ) will be

$$\frac{X(P_T/X)(G_T/X)\sigma_0\theta_A c\tau G_R \lambda^2}{2(4\pi R)^3} \quad (9)$$

### 3.2.5 Receiver Clutter Level Due to Multiple MIMO Transmitters

As noted earlier, (7) describes the clutter power at the receiver due to a *single* MIMO transmitter. When multiple MIMO transmitters are used, the *total* clutter power at the receiver (i.e., prior to DBF and MIMO Combining) will be the result of the incoherent summation of contributions from all of the various MIMO transmitters. The expected power due to this summation will therefore be  $X$  times larger than (7), i.e.,

$$CP_{MIMO} = \frac{X(P_T/X)(G_T/X)\sigma_0 e\theta_A c\tau G_E \lambda^2}{2(4\pi R)^3} \geq CP_{ABF} \cdot \left( \frac{e}{XN} \right) \quad (10)$$

Of course, the receiver noise power will be  $kT_0FB$ . Thus, the CNR at the receive will be equal to  $[equation (10)] / kT_0FB$ . However, in comparing a conventional receiver's CNR to a MIMO receiver's CNR, one must use care. In the MIMO case, the receiver bandwidth,  $B$ , can be much larger than any individual signal's bandwidth,  $B_s$ . There is no requirement that the various MIMO transmitters all utilize the same frequency band—in fact, there may be certain advantages to not using the same frequency band, as well as disadvantages, such as the potential loss of target coherence.} Two cases are considered below.

*Case 1:* Suppose all MIMO transmitters share the *same* frequency band. As a result, the receiver bandwidth will likely be chosen (as in conventional radar) such that  $B \approx B_s$ . Hence, we may compare (10) to (4) directly. This reveals that there will be a large reduction in clutter power when MIMO is used along the vertical axis of the array, resulting in  $X$  times less clutter power. However, the clutter power will not be reduced at all when MIMO is used along the horizontal axis of the array.

*Case 2:* Suppose each of the MIMO transmitters utilizes a unique band of size  $B_s$  (i.e., our MIMO mode employs “FDMA-like” waveforms). If these frequency bands are adjacent, and we are using a single receiver to receive all  $X$  MIMO transmissions, then we would have  $B \geq X \cdot B_s$ . As a result, the CNR at the output of the receiver would be at least  $X$  times smaller than the case where  $B \approx B_s$ . That is, the total clutter to noise at the output of the receiver would be reduced by a factor of  $e/X^2$  as compared with conventional ABF.

Of course, this reduction in CNR would be achieved at the cost of requiring a higher speed A/D converter. Since A/D dynamic range usually decreases as the sampling rate increases, architectures wherein  $B \geq X \cdot B_s$  may be counterproductive. However, it is still possible to reap the benefit of reduced CNR without necessitating high speed A/D converters. The idea is illustrated in Figure 10b. Here, all MIMO transmit signals are separated via filters prior to entering the sensitive receiver components. Consequently, each receiver processes clutter from only a single MIMO transmitter (note that such a system would require  $X \cdot N$  receivers). The total clutter power at the receiver is then given by (7)<sup>8</sup>. Note that for most waveforms, separating the MIMO transmit signals prior to receiver processing is not practical. However, in the special case where FDMA (frequency division multiple access) waveforms are used, the separation is indeed easy (although additional receivers are required). In comparing (7) to (4), note that each receiver will be subjected to clutter that is reduced by a factor of  $1/X$  to  $1/X^2$  depending aperture is partitioning (i.e., the value of  $e$ ).

---

<sup>8</sup> Any device in the receive chain prior to the analog filter bank (e.g., amplifiers used to compensate for losses in the filters) will need to deal with the total clutter power in (10), along with the associated dynamic range and isolation levels. It is recognized that such components might raise the overall noise figure of the receiver; more study is needed. The remainder of this report does not address this potential increase in noise figure.

### 3.2.6 Clutter Power Summary

Table 3 summarizes the relationship between CNR and hardware configuration (i.e., type of transmitter beamforming). All power levels are referenced to a noise power level of  $kT_0FB$ .

**TABLE 3**

**Reduction in CNR as compared with the CNR (after beamforming) of a conventional ABF**

$$\text{radar, } P_t G_t \sigma_0 \theta_A c \tau G_R \lambda^2 / 2 (4\pi R)^3 kT_0 FB$$

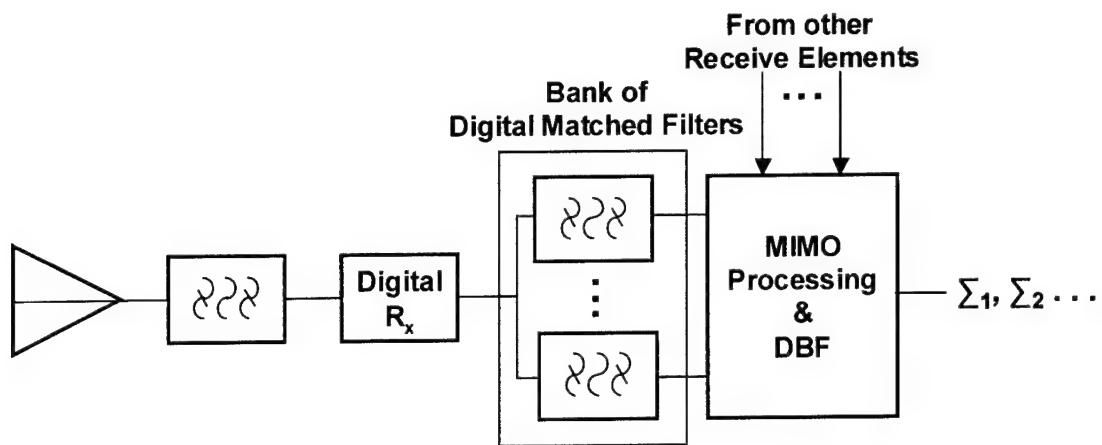
Location in Receive Chain			
Type of Transmitter		<b>CNR Reduction Factor at the Input to each Receive Element</b>	<b>CNR Reduction at Output of Beamformer</b>
	<b>Focused Beam</b>	$G_E/G_R \geq 1/N$	1
	<b>Spoiled Beam</b>	$eG_E/XG_R \geq e/XN$	$e/X$
	<b>MIMO</b>	Std. Rx: $eG_E/XG_R \geq e/XN$ FDMA Filtering Rx: $eG_E/X^2G_R \geq e/X^2N$	$1/X$

### 3.3 RECEIVER DYNAMIC RANGE

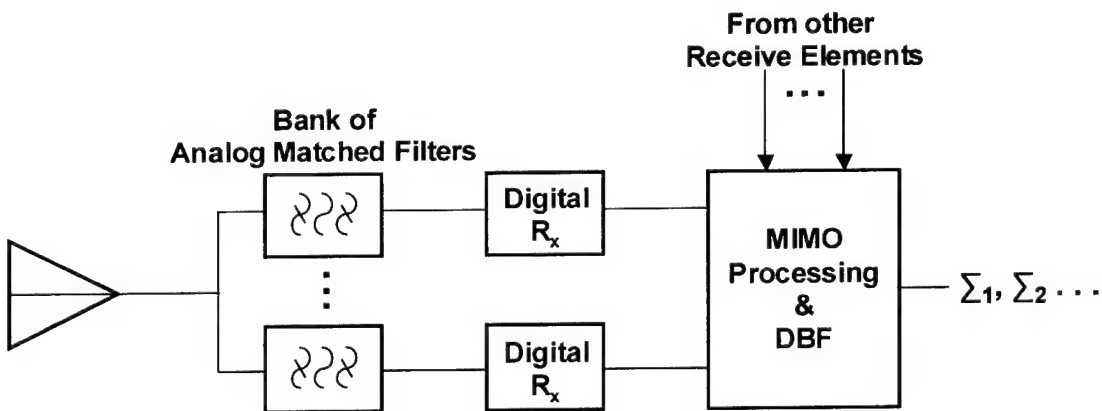
The term *dynamic range* describes how well a system handles signals of varying power levels. Unfortunately, the term can be somewhat confusing because it has been defined in many different ways. As it relates to radar sensitivity, the term “instantaneous dynamic range” is often used in discussions relating to the ability of the radar to detect a weak target signal in the presence of strong interference. “Instantaneous dynamic range” relates to the system’s performance in the presence of both strong and weak signals (at the same time). This should not be confused with other terms, such as “gain controlled dynamic range,” wherein strong and weak signals are presented one at a time.

From a radar sensitivity perspective, then, we shall the define “instantaneous dynamic range” as:

$$DR \triangleq \frac{\text{Power of strongest signal(s) that can be detected without generating distortion that is detectable}}{\text{Power of weakest signal that is properly detected}}$$



(a)



(b)

Figure 10. MIMO receiver architectures. (a) Digital MIMO filters, (b) Analog MIMO filters.

Oftentimes, the strongest signal that the receiver must handle is backscattered clutter (note: this is often the case of interest in Navy surface TAMD radar, which is the primary application that we will consider here). Furthermore, the weakest signal of interest is a target at the thermal noise level of the receiver (i.e., noise-limited detection). **Under such conditions, the required dynamic range should exceed to the CNR at the receiver so as to prevent receiver saturation.** Thus, for traditional analog beamforming radars, the dynamic range requirement is

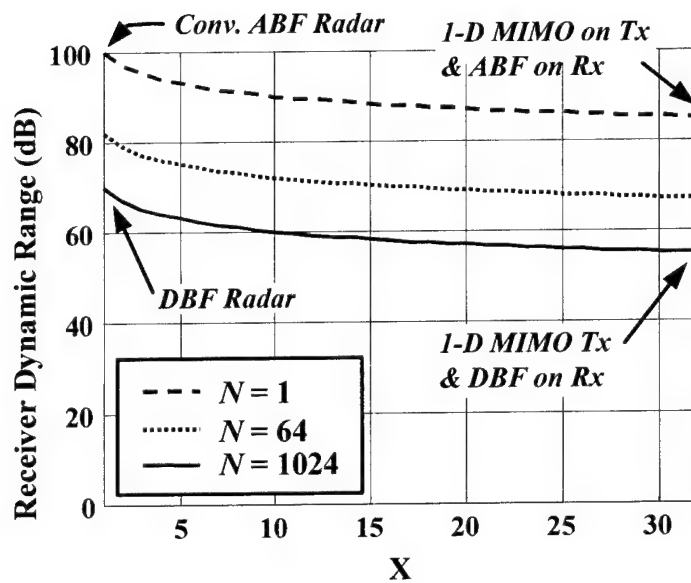
$$DR > \frac{CP_{ABF}}{kT_0FB} = \frac{P_T G_T \sigma_0 \theta_A c \tau G_R \lambda^2}{2(4\pi R)^3 kT_0FB}.$$

For DBF radars,, we require  $DR > CP_{DBF}/kT_0FB$ , which represents a reduction by a factor of  $G_E/G_R \geq 1/N$  as compared with conventional ABF radar. Moreover, time-energy management techniques can be used to reduce the receiver dynamic range requirement even further! For transmit beamspilling and/or MIMO, we require only  $DR > CP_{BS}/kT_0FB = CP_{MIMO}/kT_0FB$ , which is a reduction of  $eG_E/XG_R \geq e/XN$  as compared with conventional ABF. For FDMA MIMO, the required receiver dynamic range is even lower – only  $DR > CP_{M1}/kT_0FB$ , which is  $eG_E/X^2G_R \geq e/X^2N$  lower than traditional ABF. Note that in each case, the reduction in dynamic range mirrors the reduction in CNR, as summarized in Table 3.

To illustrate the benefits of time-energy management, let us consider an example. Suppose a conventional analog beamforming (ABF) radar contains a  $32 \times 32$  aperture (i.e., 1024 elements) and requires an instantaneous receiver dynamic range of 100 dB so as to deal with clutter. Furthermore, suppose MIMO transmitters are installed in one dimension, along the vertical axis of the array face, effectively spoiling the transmit beam in elevation. With ABF followed by a digital MIMO receiver (as in Figure 10a), Figure 11 (top) shows that the required dynamic range is thus reduced to about 85 dB. In contrast, by applying full digital beamforming on receive (without MIMO on transmit), this figure shows the required dynamic range is reduced to about 70 dB. Lastly, by digitizing on *both* transmit (i.e., MIMO) and receive (i.e., DBF), we see the dynamic range is reduced to 55dB.

As a second example, suppose that FDMA MIMO waveforms are used, along with banks of analog filtering MIMO receivers (as in Figure 10b). With MIMO on transmit and ABF on receive, Figure 11 (bottom) shows that the required dynamic range is reduced to about 70 dB. This matches the performance of pure DBF, while requiring fewer receivers! Moreover, by digitizing on both transmit (i.e., MIMO) and receive (i.e., DBF), this plot shows the dynamic range is reduced to about 40 dB.

(a)



(b)

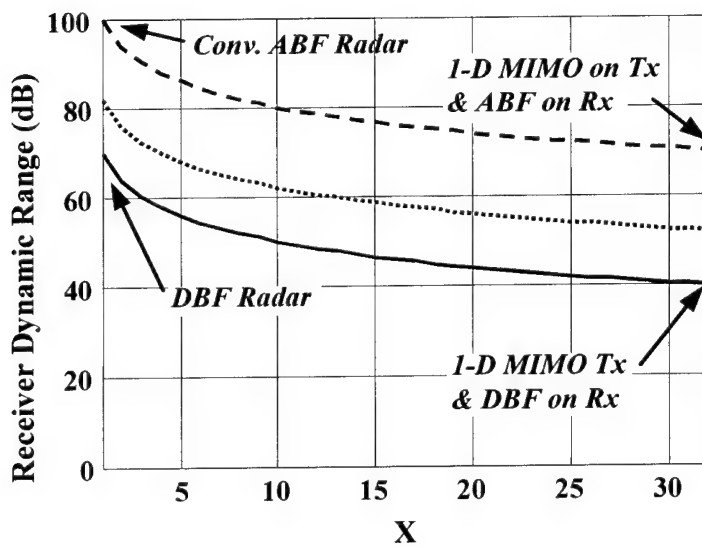


Figure 11. Examples illustrating the reduction in dynamic range achieved via beamspooling and/or MIMO.  
(a) 1-D MIMO, or 1-D Beamspooling. (b) 1-D MIMO with FDMA filterbank receiver(s).

As further examples, we might consider installing MIMO (or FDMA MIMO) transmitters along two-dimensions of the array (e.g., we could partition the transmit aperture both vertically and horizontally, and transmit an orthogonal waveform from each of the resulting subarrays). In doing so, the reduction in dynamic range would be as shown in Figure 12.

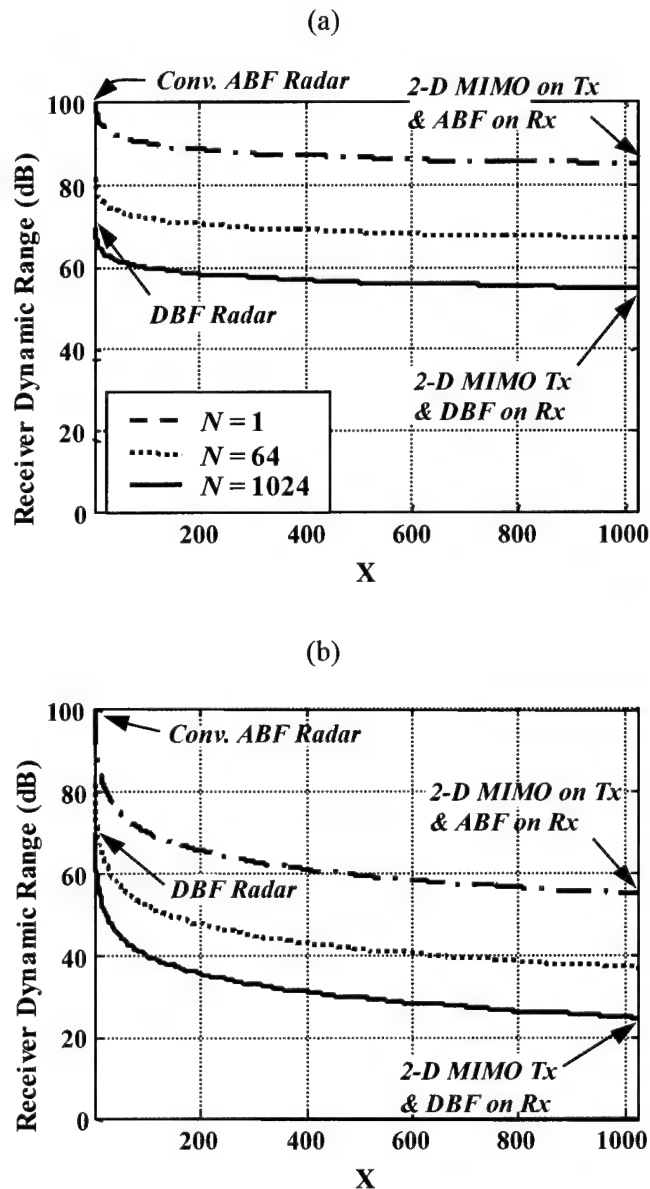


Figure 12. Examples illustrating the reduction in dynamic range achieved via beamspiling and/or MIMO.  
(a) 2-D MIMO, or 2-D Beamspiling. (b) 2-D MIMO with FDMA filterbank receiver(s).



### 3.4 RECEIVER ISOLATION

Radar designers also must worry about isolating their receivers from their transmitters. In fact, two interrelated isolation issues must be considered when specifying radar requirements. *First* – isolation must be sufficient so as to keep the receiver from saturating on strong, short-range clutter. Ideally, the transmitted signal would be attenuated to levels below thermal noise at the receiver (although nulling techniques have been used to cancel higher levels of interference). *Second* – isolation must be sufficient so as to keep the receiver from saturating on the transmitted signal itself (which can enter the receiver through a number of mechanisms, such as VSWR and mutual coupling).

Radars typically deal with the first problem by (1) employing high dynamic range receivers, and/or (2) employing sensitivity time control, and/or (3) varying the PRF. As we saw in Chapter 3.3, both beamspoiling and MIMO effectively reduce the strength of the short-range clutter<sup>9</sup>. As a result, they will also decrease the related isolation requirement by  $e/XN$  to  $e/X^2N$ .

As for the second problem, most radars isolate their receivers from their transmitters by employing pulsed waveforms and duplexers (or similar devices). As such, the receivers are never ‘listening’ while the radar is transmitting, thereby avoiding saturation on the transmitted signal. However, achieving the necessary isolation is a more difficult problem for CW radars (i.e., radars that simultaneously transmit and receive – including radars that employ arbitrary waveforms).<sup>10</sup> To isolate the receivers from the

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<sup>9</sup> Note that for above-horizon search, a focused beam might still be used as long as a low sidelobe aperture shading (in the direction of the short-range clutter) is employed. MIMO transmit channels, if chosen, might also employ this trick to increase isolation. With beamspoiling, however, this trick is not always as useful due to the difficulty in controlling the sidelobes of a spoiled beam – at least one spoiled using phase-only shading techniques.

<sup>10</sup> This is unfortunate since CW operation offers many potential benefits. For example, CW radar transmitters can be less costly and complex since they may operate at lower voltages, with less energy storage. Furthermore, high-power switches can be avoided. Best of all, a single CW-capable system can perform multiple, diverse functions (e.g., radar, communications, and electronic warfare) while employing whatever waveforms are desired.

transmitted signal itself, CW systems are usually designed to use physically separated transmit and receive antennas (while introducing absorbers between and around the antennas to increase isolation, if needed)<sup>11</sup>. Ideally, the transmitted signal would be attenuated to levels below thermal noise at the receiver (although nulling techniques can be used to cancel higher levels, as long as the receiver does not saturate).

Achieving the necessary isolation by such means can be challenging. Fortunately, MIMO can help here as well. As described earlier, an FDMA MIMO system could be designed to employ analog filters (to separate the MIMO transmit signals) prior to each receiver. The filters thus act to isolate each receiver from all but one MIMO transmit signal, thereby reducing the necessary isolation levels. In fact, it might even be possible to use a single array for both transmit and receive. With such an array, the worst-case isolation requirement at each antenna element is associated with the receiver that is matched to the frequency band of the signal being transmitted at that element. Hence, one might simply omit this receiver from the bank of  $X$  receivers at each element. This results in a system with  $X \cdot N - X$  receivers, and a marginal reduction in gain but a great benefit in isolation.<sup>12 13</sup>

### 3.5 TRANSMITTED SPURS

For a number of different reasons, spurious (i.e., undesirable) signals can arise during the creation and/or transmission of a waveform. This is true for radars, communication systems, and for just about

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<sup>11</sup> One disadvantage of employing separate transmit and receive arrays is that this approach results in an effective loss of 3 to 6 dB (since the combined aperture, operating in a pulsed fashion, would have 3 dB more gain – and possibly more power if *all* elements could transmit). Secondly, many radar platforms are not big enough to have two large antennas (e.g., small unmanned air vehicles).

<sup>12</sup> Since other nearby transmit elements could cause isolation problems, this might need to be done for the bands corresponding to these transmitters too.

<sup>13</sup> Alternately, a band-stop filter could be used prior to the receiver in Figure 10b. The stopband would be tuned to the transmit frequency of the element.

any other RF system<sup>14</sup>. Amplifiers, for example, produce harmonics as well as in-band and adjacent-band spurs. Waveform generators, such as Direct Digital Synthesizers, generate spurs due to varying mechanisms including phase truncation and quantization.

Transmitted spurs can cause a number of serious problems. For example when transmitted, such spurs can interfere with radar detection. They can also cause EMI that is harmful to other RF systems (consequently, as the RF spectrum has become more heavily utilized, interest in EMC has steadily grown). As a result, RF designers must be quite careful to control the spurious signals that arise within the transmitter chain.

To make matters worse, the spurious signals emitted by one transmit module can sometimes be correlated with the spurs transmitted by from other modules across the array. Consequently, these spurious signals may experience array *gain* during transmit beamforming.

In light of this fact, reducing beamformer gain (e.g., through beamspoiling) can thus be of some help. Better still, when the source of the spur is waveform related, MIMO can offer a great benefit. Since the MIMO transmit waveforms are orthogonal, many spurs (e.g., those due to phase truncation and quantization in DDSs) will not be coherent across the array. As a result, these spurs will not be subject to array gain on transmit, thereby reducing interference – both to the radar itself and to other RF systems.

### 3.6 PHASE NOISE, STABILITY, AND CLUTTER REJECTION

To detect targets in strong clutter, filters (such as MTI or pulse-Doppler filters) are used to cancel the clutter. Clutter rejection requirements are thus determined by expected input target and clutter levels, as well as the coherence of these signals throughout the dwell. System stability requirements [6] are thus driven by the need to keep residual clutter levels (after cancellation) from competing with the target. In this section, we quantify the benefits of time-energy management towards clutter rejection and system stability.

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<sup>14</sup> This problem is notoriously bad in radar, due to the nonlinear high-power amplification modes that are usually employed.

Let us begin by defining a commonly used figure of merit, the Clutter Improvement Factor (CIF). CIF is usually defined as the “Signal-to-Clutter Improvement Factor” necessary to detect some desired target. That is:

$$CIF \triangleq (S_{out}/C_{out})/(S_{in}/C_{in}). \quad (11)$$

Note that a radar’s CIF is usually defined after all coherent processing, with the worst-case CIF requirement generally established when the radar’s beam is pointed toward clutter (resulting in the largest  $C_{in}$ ).

For our purposes, it is convenient to re-write (11) as:

$$CIF \triangleq \frac{S_{out}/C_{out}}{S_{in}/C_{in}} = \frac{S_{out}}{S_{in}} \cdot \frac{C_{out}}{C_{in}} = \underbrace{\frac{SNR_{out}}{SNR_{in}}}_{\text{SNR Gain}} \cdot \underbrace{\frac{CNR_{in}}{CNR_{out}}}_{\text{CNR Improvement Factor}}$$

As we shall see, the rightmost expression is quite convenient for studying the impact of time-energy management techniques. This expression contains two terms. The first term represents the SNR gain (including scalloping losses) that may occur in the MTI/Doppler system. The second term represents the CNR improvement factor (a.k.a. the MTI Improvement Factor).

For surface-based radar systems, clutter rejection and system stability requirements are usually defined assuming the target lies in a clear region of Doppler space. In this region, target detection would ideally be limited only by target’s SNR after processing (i.e., not clutter). Hence, our criteria for specifying clutter rejection and stability levels is that the residual clutter (after MTI/Doppler processing) must be no greater than the thermal noise power in each Doppler filter corresponding to targets in the clear region.

With clutter assumed to be in the target’s Doppler sidelobes, it is clearly the CNR improvement factor that drives system stability requirements. If the system hardware supports a CNR improvement factor greater than the single-pulse CNR, clutter can be cancelled below noise (e.g., via MTI filtering or low Doppler sidelobes).

For example, one important radar hardware specification is that of “one-sided phase noise.” Assuming a flat noise floor and ignoring range ambiguous clutter, the system-level phase noise

requirement can be shown to depend directly on the CNR Improvement Factor (not CIF!). The relationship is

$$S \triangleq \frac{CNR_{out} \cdot 2}{CNR_m \cdot \rho \cdot B} \quad (12)$$

where  $B$  is the received bandwidth, the factor of 2 in the numerator represents specifying a one-sided phase noise measurement, and the factor of  $\rho$  in the denominator allows for correlation between the received signal and the mixing LO (if present). In practice, due to the potential for multiple instabilities, we would usually allow some margin by designing each component for a smaller  $S$ .

Note that (12) implies the system level phase noise requirement is driven by the CNR improvement factor; it does not depend at all on the SNR gain factor at all. *Hence, any scheme by which we can reduce the single pulse CNR – even if it also reduces single pulse SNR and thereby forces us to integrate longer – will improve system stability.* This observation is the key to exploiting time-energy management techniques! For example, Table 3 says that beamspoiling can reduce the single-pulse CNR by  $e/X$ . MIMO reduces the single pulse CNR by  $1/X$ . Note that in both cases, the target SNR will be reduced as well. In fact, the target SNR could be reduced by even more than the CNR. Nonetheless, the loss in SNR can be erased by integrating longer – without increasing the system level phase noise requirement. Moreover, even though we may need to integrate longer in each beam position, the total Search Frame Time can be maintained by forming multiple simultaneous beams on receive. Table 4 summarizes the improvements in phase noise resulting from beamspoiling and/or MIMO.

Note that other interpulse stability requirements are tabulated in [6]. In some cases, these requirements will depend on the CNR Improvement Factor, not the CIF, and can thereby be eased through time-energy management.

### 3.6.1 Phase Noise Example

Suppose a conventional radar has  $B = 1\text{MHz}$ , and a single pulse CNR of 80 dB (at some range of interest). For noise limited detection, (12) requires an equivalent flat oscillator phase noise of  $-140\text{ dBc/Hz}$ . However, if we use MIMO on transmit, we saw earlier that the CNR after beamforming drops as  $1/X$ . Furthermore, note that it does not matter how the MIMO transmitters are aligned in 1-D (i.e., horizontally vs. vertically) or 2-D. Consequently, the system level phase noise requirement is also reduced as the number of MIMO transmitters is increased, as shown in Figure 13. Note that with  $X = 1000$  transmitters, the single pulse CNR drops to 50dB, and the phase noise requirement drops to  $-110\text{ dBc/Hz}$ .

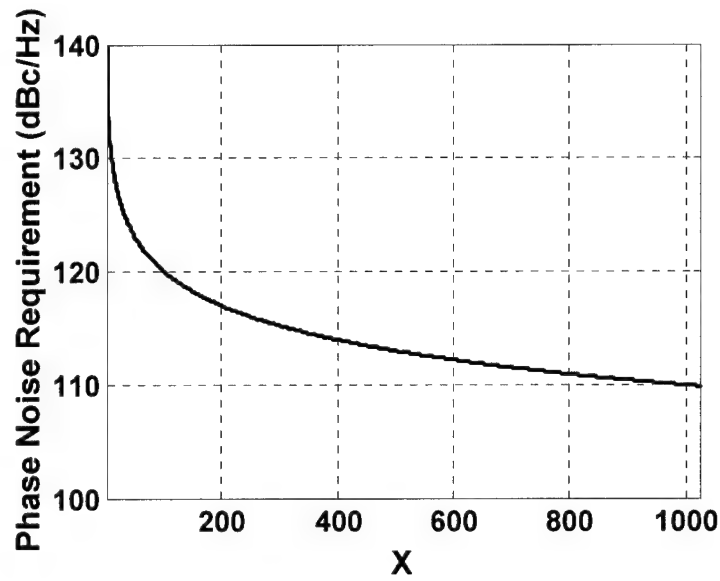


Figure 13. System phase noise requirement vs. number of MIMO transmitters.

TABLE 4

Reduction in system phase noise requirement, as well as certain other sources of  
interpulse instability

Type of Transmitter	Instability Reduction Factor
Focused Beam	1
Spoiled Beam	$e/X$
MIMO	$1/X$

### 3.6.2 Time-Energy Management and Clutter Nulling

It is also worth noting that, since the single-pulse CNR can be reduced (with MIMO or beamspoiling), the performance of the clutter rejection filter itself can be improved when compared to the

filter that would be required in a conventional radar. In the case of MTI filtering, the clutter notch can be made less deep. This, in turn, allows the notch to be sharper, thereby improving overall MDV. In the case of Doppler filter-based clutter rejection, MIMO uses a dwell that is  $X$  times longer to compensate for its reduced transmit gain. Thus, the MIMO radar's Doppler FFT filter will have a sharper mainlobe (i.e., improved Doppler resolution, improved MDV). Moreover, the requirement on Doppler sidelobes levels is less stressing than in a comparable conventional radar

### 3.7 CONCLUDING REMARKS

This section has derived expressions for the distributed clutter levels expected at various points in a radar receive chain. At the receiver, it was shown that  $X:1$  beamspoiling can reduce the clutter by a factor of up to  $1/XN$  where  $N$  is the number of digital receivers. Moreover, by using  $X$  MIMO transmitters, clutter at the receiver can be reduced by a factor of  $1/XN$  to  $1/X^2N$ .

These reductions in clutter were shown to lead to reductions in receiver dynamic range. The reduction in dynamic range was shown to be as high as  $1/X^2N$ . Furthermore, MIMO digitization on transmit was shown to offer benefits analogous to DBF on receive, and combined operation (i.e., MIMO digitization on transmit plus DBF on receive) was shown to offer the greatest benefits of all.

The reductions in clutter power were also shown to lead to reduced isolation requirements for strong, short-range clutter. MIMO architectures were further described that greatly reduce transmitter isolation requirements.

It was also noted that utilizing orthogonal transmit waveforms prevents some transmitter spurs from being correlated across the array, thereby receiving array gain. This helps in designing systems for EMC, and may also potentially reduce in-band self-interference.

Single pulse CNR levels after beamforming were also defined. It was further shown that time-energy management can reduce these CNR levels by up to  $1/X$ . It was argued that a number of hardware stability factors, such as phase noise, are critically tied to the single pulse CNR level. Hence, the benefits of time-energy management techniques like MIMO can include improved system stability and clutter rejection.

Although expressions derived in Chapter 1 pertain to surface-based radars and surface clutter, many of the benefits of time-energy management apply to other scenarios as well. For example, in airborne radar the effective size of the uncompressed clutter patch is  $\sigma_0 \cdot \theta_A \cdot R \cdot c\tau/2 \cdot \sec\psi$  where  $\psi$  denotes the

grazing angle. This is the same as the clutter patch term in (3), except for the presence of  $\sec\psi$ . Since the clutter patch size does not depend on  $\theta_E$ , beam spoiling and/or MIMO will reduce CNR in a manner directly analogous to that seen in Chapter 1.

Likewise, beam spoiling and/or MIMO can also be used to reduce volumetric clutter levels. With volumetric clutter, the effective size is  $\eta \cdot \omega R^2 \cdot c\tau/2$  where  $\eta$  is the cross section per unit volume, and  $\omega$  is the solid angle of the radar beam (in steradians) or the portion thereof occupied by the volumetric clutter. If this clutter is distributed across the entire radar beam, we would have  $\omega = (\theta_A \cdot R) \cdot (\theta_E \cdot R)$  and there would be no reduction in CNR due to spoiling or MIMO. However, if the clutter is more locally distributed such that  $\omega < (\theta_A \cdot R) \cdot (\theta_E \cdot R)$ , a reduction in clutter power can result.

Finally, note that this chapter did not specifically distinguish between beamspoiling and machine gunning. At the level of the analysis in this chapter, machine gunning radars enjoy benefits similar to that of beamspoiling systems. Moreover, in complex environments (such as those containing propagation ducts) it seems reasonable that the additional spatial transmit energy control provided by machine gunning systems could be used to improve performance beyond that of beamspoiling systems.



## 4. LOW PROBABILITY OF INTERCEPT RADAR OPERATION

### 4.1 MOTIVATION

As mentioned in Chapter 1, military radars are often designed so as to produce very high peak power levels (i.e., high “Effective Isotropically Radiated Power” levels – a.k.a. “EIRP”). This, in turn, enables detection of the small targets.

Unfortunately, high EIRP emissions can often be detected by unintended recipients. In some cases, the unintended recipient does not desire the intercept at all. In this case, the radar’s high EIRP is said to have resulted in EMI (electromagnetic interference) – a potentially serious problem with Navy fleet radars today.

In other cases, the unintended recipient is actually an adversary attempting to locate the radar and/or exploit its signals. To do this, the adversary employs a special Electronic Intelligence (ELINT) receiver, which processes received signals in such a way as to increase the probability of intercepting the radar signals. In fact, a good ELINT system can usually detect a *conventional* radar at much longer range than the radar can detect the platform carrying the ELINT receiver. This, of course, is due to the added propagation loss the radar experiences as it operates over a two-way path, while the ELINT receiver only operates over a one-way path.

Fortunately, time-energy management techniques, along with other measures, can be used to reduce EMI and lower the probability of intercept by an adversary’s ELINT system. Radars that employ such measures are known as LPI (Low Probability of Intercept) radars.

Typically, LPI radars differ from conventional radars in that their peak radiated power density has been reduced. For example, reductions can be achieved temporally by using long pulses (or CW modes of operation) instead of short pulses. Reductions can also be achieved by spreading the radar signal energy over a large spectrum (e.g., by using spread spectrum techniques). Finally, reductions can be achieved by reducing the transmit gain (e.g., via ubiquitous radar operation, MIMO radar operation, and/or beamspoiling on transmit).

In the following sections, we analyze how time-energy management techniques can be used to reduce the probability of the radar being intercepted by an enemy's ELINT system.

## 4.2 RADAR INTERCEPT RANGE WHILE IN MIMO-MODE

To assess the impact of time-energy management techniques on radar interceptability, let us begin by defining target SNR (at the radar). Considering the radar system in Table 2, the target's power after pulse compression, receive beamforming and MIMO processing (i.e., transmit beamforming performed at the receiver) will be:

$$\underbrace{\frac{P_T}{X} \frac{G_T}{X} \left[ \frac{1}{4\pi R^2} \right]}_{\text{Effective Tx power per unit area (single Tx) at the range of target}} \underbrace{\sigma_t}_{\text{Target RCS}} \underbrace{\left[ \frac{1}{4\pi R^2} \right] \frac{G_R \lambda^2}{4\pi}}_{\text{Amount of reflected signal captured at the receiver}} \underbrace{G_M B_S \tau}_{\text{Processing Gain}}. \quad (13)$$

In contrast, the receiver noise power is  $kT_0FB$ . Hence, the SNR of the target after pulse compression, receive beamforming and MIMO processing (if applicable) will be:

$$SNR_t = \frac{P_T G_T \sigma_t G_R \lambda^2 G_M B_S \tau}{X^2 (4\pi)^3 R^4 kT_0 F B}. \quad (14)$$

Will the radar be able to detect this target before it is itself detected by the enemy's ELINT system? To answer this question, let us assume the enemy's ELINT system is characterized by the parameters defined in Table 5. Under these assumptions, the radar's power (at the ELINT receiver) after processing is:

$$\underbrace{\frac{P_T}{L} \frac{G_T}{X} \left[ \frac{1}{4\pi R_e^2} \right]}_{\text{Total effective radar Tx power per unit area at the ELINT range}} \underbrace{\frac{G_{e,r} \lambda^2}{4\pi}}_{\text{Amount of reflected signal captured at the receiver}} \underbrace{\left( \lfloor B_e \rfloor \lfloor \tau_e \rfloor \right)^{\gamma}}_{\text{Processing Gain}}, \quad (15)$$

and the noise power at the ELINT receiver will be  $kT_0 F_e B_e$ . In (15),  $\lfloor B_e \rfloor$  denotes the subband of  $B_e$  which overlaps with the radar transmit band, and  $\lfloor \tau_e \rfloor$  denotes the interval within  $\tau_e$  which overlaps with

received radar emissions. As such, the radar's SNR at the ELINT receiver (after ELINT processing) will thus be:

$$SNR_e = \frac{P_T G_T G_{e,r} \lambda^2 (\lfloor B_e \rfloor \lfloor \tau_e \rfloor)^\gamma}{L X (4\pi R_e)^2 k T_0 F_e B_e} \quad (16)$$

**TABLE 5**  
**ELINT parameters**

SYMBOLS	DEFINITION
$G_{T,e}$	Radar transmit gain in direction of the ELINT system
$G_{e,r}$	ELINT antenna gain in direction of radar
$B_e$	ELINT receiver's processing bandwidth
$F_e$	ELINT receiver's noise figure
$L$	Loss incurred if ELINT band is not large enough to capture entire radar transmit spectrum.
$\tau_e$	Noncoherent integration time of ELINT processor
$R_e$	Range of ELINT with respect to radar transmitter
$D_e$	ELINT detection threshold
$\gamma$	Noncoherent processing factor

For the radar to detect the target, we require  $SNR_t > D_t$ . Likewise, for the ELINT system to detect the radar, we require  $SNR_e > D_e$ . Assuming both systems use thresholds set for similar probabilities of detection and false alarm,  $D_t \cong D_e$ . Equal probability of detection, then, occurs when the various SNRs are about equal, i.e., when  $SNR_t = SNR_e$ . From (14) and (16), the condition for equal probability of detection is

$$\frac{P_T G_T \sigma_t G_R \lambda^2 G_M B_S \tau}{X^2 (4\pi)^3 R^4 k T_0 F B} = \frac{P_T G_T G_{e,r} \lambda^2 (\lfloor B_e \rfloor \lfloor \tau_e \rfloor)^\gamma}{L X (4\pi R_e)^2 k T_0 F_e B_e} ,$$

or simply

$$\frac{\sigma_t G_R G_M B_S \tau}{X 4\pi R^4 F B} = \frac{G_{e,r} (\lfloor B_e \rfloor \lfloor \tau_e \rfloor)^\gamma}{L R_e^2 F_e B_e} . \quad (17)$$

Assuming ideal MIMO combining such that  $G_M = X$ , we may rearrange (17) a bit so that we have

$$\frac{R^4}{R_e^2} = \frac{L G_R}{G_{e,r}} \cdot \frac{\sigma_t}{4\pi} \cdot \frac{F_e}{F} \cdot \frac{B_e}{B} \cdot \frac{B_S \tau}{(\lfloor B_e \rfloor \lfloor \tau_e \rfloor)^\gamma} . \quad (18)$$

This equation can be further simplified if we assume the ELINT system has an omnidirectional antenna ( $G_{e,r}=1$ ) and a well designed LNA-based receiver (so that  $F_e \approx F$ ). Furthermore, the radar's receiver bandwidth is usually matched to its signal, such that  $B \approx B_S$ . Consequently, (18) reduces to:

$$\frac{R^4}{R_e^2} = L \cdot G_R \cdot \frac{\sigma_t}{4\pi} \cdot B_e \cdot \frac{\tau}{(\lfloor B_e \rfloor \lfloor \tau_e \rfloor)^\gamma} . \quad (19)$$

Choosing  $R=R_e$ , we can solve for the range associated with “equal detectability.” That is, the range within which the radar holds an advantage. (Beyond this range, the ELINT system has the advantage.) This range is equal to:

$$R = \left[ L \cdot G_R \cdot \frac{\sigma_t}{4\pi} \cdot B_e \cdot \frac{\tau}{(\lfloor B_e \rfloor \lfloor \tau_e \rfloor)^\gamma} \right]^{1/2} . \quad (20)$$

To understand how MIMO (and other techniques) effect this intercept range, it is helpful to plug some values into (20). This is done in section 4.4. However, before we do this, first a few words about other time-energy management techniques.

### 4.3 RADAR INTERCEPT RANGE WHILE USING TRANSMIT BEAMSPOILING OR MACHINE GUNNING

For a conventional single-beam radar, note that the term  $G_T/X$  would be replaced by  $G_{T,e}$  in (16) and subsequent equations. One advantage of conventional single-beam radar operation is that  $G_{T,e}$  can be made low when the ELINT system lies within the radar's transmit sidelobes. Unfortunately, conventional radars are much more detectable when the ELINT system is in the radar's transmit mainlobe. For a radar that performs volume search, the main beam must sweep across all angles; consequently, when the main beam points toward the ELINT system, a conventional single-beam radar is easy to detect.

In contrast, transmit beamspiling radars have a reduced mainlobe transmit beamforming gain. Consequently, transmit beamspiling should help provide LPI operation similar to MIMO operation considered in Section 4.2.

Finally, in a machine gunning radar system, note that the peak mainlobe transmit power is not reduced. However, the subpulse duration will be shorter. This, in turn, can possibly lead to improvements in LPI as indicated by the equations above.

### 4.4 LPI EXAMPLE

To illustrate the potential LPI benefits resulting from time-energy management, let's examine the detectability of hypothetical multifunction military radar system. Today, a typical naval surface-based radar might employ a large aperture ( $G_T = G_R = 1000$ ). In search modes, this radar might employ short pulses ( $\tau = 2$  to 10 usec) of moderate bandwidths ( $B = 1$  MHz)<sup>15</sup>. Pulses are often coherently processed using pulse-Doppler radar techniques. Let's assume the system operates with a medium PRF (5 kHz), and with a typical dwell lasting about 10 pulses ( $\tau_i \geq 20$  usec).

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<sup>15</sup> In other modes, the radar's waveform bandwidth might be higher, e.g., to support fire control. Thus, the ELINT system designers may not wish to optimize the ELINT's processing bandwidth for the narrow radar search modes.

In contrast, an ELINT system would typically employ a wider receiver bandwidth, with signals filtered into subbands prior to noncoherent integration. Let us assume the ELINT's processing bandwidth is  $B_e = 50\text{MHz}$ , resulting in the likelihood that  $\lfloor B_e \rfloor \approx B_s$ . Furthermore, assume the integration time is chosen to be longer than a typical radar pulse (e.g.,  $\tau_e \approx 6\text{ usec}$ ). Moreover, assume the noncoherent integration is characterized by  $\gamma = .65$ . Finally, assume the ELINT system resides aboard an attacking aircraft having 0 dBsm cross section ( $\sigma_t = 1$ ). Substituting these values into the intercept range analysis above, we find that the ELINT system will hold the advantage when  $R > 225\text{ m}$ .

However, suppose we instead design the radar to be MIMO-capable ( $X = 1000$ ) while maintaining comparable sensitivity. To maintain sensitivity, (14) suggests the dwell time will need to be increased by a factor of  $X$ , yielding  $\tau_i \geq 20\text{ msec}$ . As a result, (20) predicts the intercept range will be

$$R = \left[ G_R \cdot \frac{\sigma_t}{4\pi} \cdot B_e \cdot \frac{\tau}{(\lfloor B_e \rfloor \lfloor \tau_e \rfloor)^\gamma} \right]^{1/2} = \left[ 1000 \cdot \frac{1}{4\pi} \cdot 50e6 \cdot \frac{.02}{(1e6 \cdot 2e-6)^{.65}} \right]^{1/2} = 7121.$$

Note that the use of MIMO has resulted in an increase in the intercept range from 225 m to 7.1 km. This is a big improvement (the intercept range is  $\sqrt{1000}$  times larger due to MIMO).

Moreover, suppose our MIMO radar is also redesigned for an increased duty factor. High duty factor radars utilize long pulses (i.e., large  $\tau$ 's and  $\tau_i$ 's), thereby allowing the radar to lower its peak power while maintaining sensitivity. In the limit, the radar might even operate in CW fashion – a possibility which is partially facilitated by MIMO as discussed in Section 3.4. Assuming both MIMO and CW operation, with power levels adjusted to maintain sensitivity, we would have  $\tau_i = 2\text{ sec}$ <sup>16</sup>. As such, (20) predicts

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<sup>16</sup> This assumes the target remains coherent throughout the dwell. For very long dwells, this is unlikely. In such cases, a combination of coherent and noncoherent integration can be used.

$$R = \left[ G_R \cdot \frac{\sigma_t}{4\pi} \cdot B_e \cdot \frac{\tau}{(\lfloor B_e \rfloor \lfloor \tau_e \rfloor)^\gamma} \right]^{1/2} = \left[ 1000 \cdot \frac{1}{4\pi} \cdot 50e6 \cdot \frac{2}{(1e6 \cdot 2e-6)^{65}} \right]^{1/2} = 71213,$$

i.e., the radar will be at an advantage whenever the target range is less than 71.2 km.

Finally, suppose our CW MIMO radar also spreads its energy over a wide frequency band. This can be done, for example, using and FDMA MIMO scheme as described in Chapter 1. With 1000 transmitters, each transmitting a 1 MHz waveform, the total band of radar emissions could be greater than 1 GHz. Assuming the ELINT system processes subbands of 50 MHz as above, then the radar will now fill the ELINT subband (and beyond), i.e.,  $\lfloor B_e \rfloor = B_e$ . However, the effective transmit power term in (15) must be reduced. In (15), a effective power of  $P_T$  was used, commensurate with incoherent combining of all MIMO waveforms. With the FDMA scheme described above, only  $1/20^{\text{th}}$  of the MIMO transmitters will be in the ELINT receive band, so the power term must be reduced to  $P_T/20$ . This adds a factor of 20 to the inner term in (20), resulting in an intercept range given by

$$R = \left[ G_R \cdot \frac{\sigma_t}{4\pi} \cdot B_e \cdot \frac{\tau}{(\lfloor B_e \rfloor \lfloor \tau_e \rfloor)^\gamma} \right]^{1/2} = \left[ 20 \cdot 1000 \cdot \frac{1}{4\pi} \cdot 50e6 \cdot \frac{.02}{(1e6 \cdot 2e-6)^{65}} \right]^{1/2} \cong 318.5 \text{ km}. \quad (21)$$

This is, of course, quite amazing! It should be noted that to get such eye-watering performance assumes successful development of (1) MIMO and FDMA transmit technology, and (2) CW radar technology – a current focus of the ONR AMRFs program. It also assumes the ELINT system would not attempt to optimize itself for MIMO radars. Clearly, this is very much a hypothetical scenario. However, the possibilities are really quite interesting.

#### 4.5 LPI SUMMARY

Beginning with simple, physics-based equations for SNR, we have derived expressions relating the interceptability of a radar to the detectability of a target. It has been shown that time-energy management techniques (like MIMO and transmit beamspiling) reduce peak radar transmit power, resulting in decreased radar interceptability. Furthermore, MIMO operation can partially facilitate CW and spread spectrum operation. Combined, these techniques can allow a fairly “large” radar to remain “quiet” over a sizeable range.

There are a number of other LPI techniques that were not discussed in this chapter. Some of these couple nicely with the MIMO concept. For example, additional LPI benefits can be achieved through bistatic radar operation. Bistatic radars allow the receive antenna to be totally silent, and therefore not detectable via ELINT. This is the subject of Chapter 7.



## 5. ANGLE ACCURACY

As compared with conventional ABF and DBF radars, DAR systems utilizing a MIMO mode can also improve the accuracy of their target angle estimates. This is most easily understood by examining the Cramer-Rao Bound (CRB) on angle accuracy for narrowband MIMO radar, and comparing it to the similar bound for conventional radar. This chapter derives the CRB for MIMO-mode DAR. In this chapter, it is assumed that both transmit and receive apertures are physically close to each other (i.e., pseudo-monostatic, if not true monostatic) and are digitized in the same way ( $X = N$ ). Note that the results can be generalized to the bistatic case where the geometry is known.

Let  $\mathbf{a}(\phi, \theta)$  represent an  $N \times 1$  array response vector for a target at direction  $(\phi, \theta)$ . Prior to pulse compression matched filtering, the received MIMO signal may be represented as

$$\mathbf{x}(t) = \alpha \mathbf{a}(\phi, \theta) \sum_{n=1 \text{ to } N} w_n(t) a_n(\phi, \theta) + \mathbf{e}(t)$$

assuming all transmitted waveforms have the same complex magnitude ( $\alpha$ ). Here,  $w_n(t)$  is the  $n^{\text{th}}$  waveform,  $a_n(\phi, \theta)$  is the  $n^{\text{th}}$  element of the array response vector and  $\mathbf{e}(t)$  is a  $N \times 1$  vector of noise at time  $t$ . Pulse compressing each receiver's output with each waveform, and then stacking the results into a vector, the signal is expressed as

$$\mathbf{x}(t) = \alpha (\mathbf{a}(\phi, \theta) \otimes \mathbf{a}(\phi, \theta)) \delta(t) + \mathbf{n}(t)$$

where  $\otimes$  is the Kronecker product and  $\mathbf{n}(t)$  is a  $N^2 \times 1$  noise vector. This signal model assumes that the cross-correlation between the waveforms is negligible and that the noise filtered through each pulse compressor is independent.

For conventional radar, the CRB for estimating the angle  $\phi$  is known to be  $\sigma_\phi^2 = \sigma^2 / 2\beta^2 \|\mathbf{d}_\phi \odot \mathbf{a}(\phi, \theta)\|^2$ , where  $\sigma^2$  is the noise power,  $\odot$  is the Hadamard product, and  $\mathbf{d}_\phi \odot \mathbf{a}$  is the derivative of the array response vector  $\mathbf{a}$  with respect to the parameter  $\phi$  (assuming  $\mathbf{a}$  is of the form  $a_n = \exp(-j\psi_n)$ )<sup>17</sup>. For our MIMO radar signal model, the effective array response vector is  $\mathbf{a} \otimes \mathbf{a}$ ; its derivative is:

$$[(\mathbf{d} \otimes \mathbf{1}) + (\mathbf{1} \otimes \mathbf{d})] \odot [\mathbf{a} \otimes \mathbf{a}]$$

When evaluating the CRB, the  $\mathbf{a}$  terms do not contribute to the norm expression in the denominator. Consequently, the CRB of MIMO radar (relative to conventional radar) is

$$\frac{\sigma_{MIMO}^2}{\sigma_{CONV}^2} = \frac{|\beta|^2 \|\mathbf{d}\|^2}{|\alpha|^2 \|(\mathbf{d} \otimes \mathbf{1}) + (\mathbf{1} \otimes \mathbf{d})\|^2} = \frac{|\beta|^2}{2N|\alpha|^2} \quad (22)$$

(Note that  $\mathbf{1}^T \mathbf{d} = 0$  when the phase center of the array is chosen as the geometric center.) For a fair comparison of MIMO and conventional radars, an equal SNR constraint requires that  $\beta = \sqrt{N}\alpha$ . Thus (22) can be simplified to  $\sigma_{MIMO}^2 / \sigma_{CONV}^2 = 0.5$ , and the estimation error (standard deviation) of the angle estimate from a MIMO radar is  $\sqrt{2}$  smaller than for a conventional radar! Intuition into this result can be gained by exploring a maximum likelihood technique for angle estimation. For this technique, the data is projected onto a grid of beams, and then the output level in each beam is compared to find the maximum. For conventional radar, the grid is formed with receive beams; representative two way (i.e., transmit and receive) beampatterns are plotted in Figure 14 as dashed lines. (Here, a Tx beam was steered to  $0^\circ$ , and Rx beams were steered to  $0^\circ$ ,  $.6^\circ$ , and  $-.6^\circ$ .) For a MIMO radar, the transmit and receive beams are moved throughout the grid together resulting in the two way patterns represented by the solid lines in Figure 14. (Here, the combined Tx-Rx beams were steered to  $0^\circ$ ,  $.6^\circ$ , and  $-.6^\circ$ .) The objective of the maximum likelihood search is to distinguish between the outputs of each beam. This objective is more easily achieved when the difference in magnitude between the beams is large. This can be seen in the

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<sup>17</sup> The CRB for estimating angle  $\theta$  is similar.

figure when looking at the beam responses at zero degrees. If a target were present at  $0^\circ$ , the difference between its response in the center beam and the other conventional beams would be about half that of the difference between the response in the center beam and the other MIMO beams.

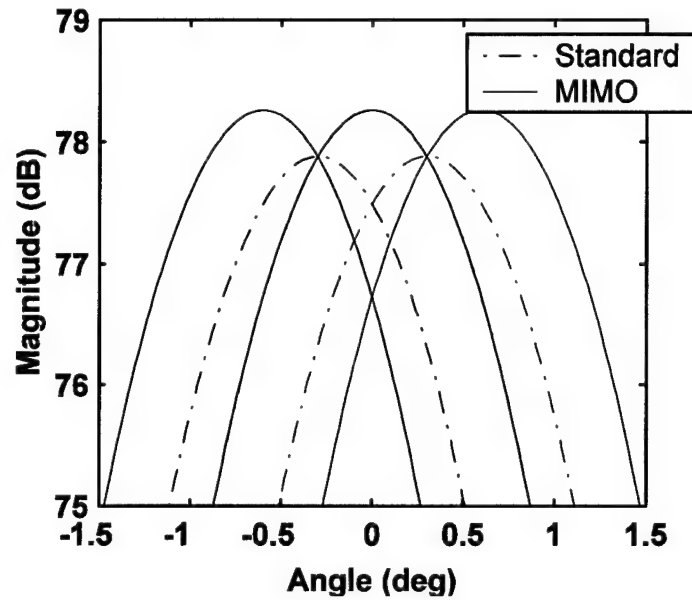


Figure 14. Two way beampatterns for conventional radar (dashed) and MIMO radar (solid) for three different beam positions.

## 6. PROCESSING CONSIDERATIONS

In a traditional DAR, the signal processor must perform a number of processing operations. Among these are two core functions, (1) Matched Filtering (i.e., to the transmitted waveform), and (2) Receive Beamforming (i.e., to combine spatial receiver channels). Note that, from the perspective of the signal processor, transmit beamforming is “free” (since signals are summed coherently in space.)

For DAR systems operating in MIMO mode, the various transmit waveforms are not coherent or focused. Transmit beamforming, then, must be effectively done on receive. Thus, the signal processor must perform three core functions, (1) Matched Filtering (i.e., to each of the transmitted waveforms), (2) Receive Beamforming, and (3) Transmit Beamforming. Mathematically, it does not matter which order is chosen for implementing these core functions. However, from an implementation perspective the order of processing can indeed effect the required processor throughput.

Conceptually, the simplest way to think of the core MIMO processing functions is to start with the received waveform:

$$\mathbf{x}(t) = \alpha \mathbf{b}(\phi, \theta) \sum_{x=1 \text{ to } X} w_x(t) a_x(\phi, \theta) + \mathbf{e}(t)$$

(with the various terms as defined in Chapter 2.4) and then matched filter to each waveform, yielding

$$\mathbf{x}(t) = \alpha (\mathbf{b}(\phi, \theta) \otimes \mathbf{a}(\phi, \theta)) \delta(t) + \mathbf{n}(t)$$

Then, we could apply an  $(X \cdot N) \times 1$  beamformer weight vector to form each output beam

$$y(t) = (\mathbf{b}(\phi, \theta) \otimes \mathbf{a}(\phi, \theta))^H \mathbf{x}(t)$$

Although conceptually simple, this approach is computationally intensive. The number of arithmetic operations per output sample is:

$$X \cdot N \cdot N_{MF} + 2X \cdot N \cdot B$$

where  $N_{MF}$  is the number taps in each matched filter (assuming the matched filters are implemented as tapped delay-line FIR filters), and  $B$  is the number of 2-way beams desired. Observe that there is also a large memory requirement, since the  $N$  receiver channels are first processed to form a much larger number (i.e.,  $X \cdot N$ ) of filtered channels prior to beamforming.

As a step toward reducing the computational load, we might consider factoring the beamforming operation into two parts: a transmit beamforming operation and a receive beamforming operation. This is possible as long as the beamformer weights can be represented as the Kronecker product of two terms, e.g.,  $\mathbf{b}(\phi, \theta) \otimes \mathbf{a}(\phi, \theta)$  above. Applying transmit beamforming first (followed by receive beamforming), the number of arithmetic operations per output sample is reduced to:

$$X \cdot N \cdot N_{MF} + 2X \cdot B_{TX} + 2N \cdot B$$

where  $B_{TX}$  is the number of transmit beams. Conversely, applying the receive beamforming first (followed by transmit beamforming), the number of arithmetic operations is reduced to:

$$X \cdot N \cdot N_{MF} + 2N \cdot B_{RX} + 2X \cdot B$$

Unfortunately, neither of these approaches has reduced the huge burden (both in terms of computation and memory) imposed by the preceding bank of matched filters!

However, in monostatic (or pseudo monostatic) configurations it is possible to reduce the processing and memory burden further by transposing the matched filtering and receive beamforming operations. Starting with the received waveform,

$$\mathbf{x}(t) = \alpha \mathbf{b}(\phi, \theta) \sum_{x=1 \text{ to } X} w_x(t) a_x(\phi, \theta) + \mathbf{e}(t)$$

we perform digital beamforming. Let  $\mathbf{B}$  represent an  $N \times B_{RX}$  matrix whose columns contain the various receive beamformer's weight vectors. Performing digital receive beamforming, we have,

$$\mathbf{z}(t) = \mathbf{B}^H \mathbf{x}(t)$$

In the monostatic (or pseudo-monostatic) configuration, the matched filtering and transmit beamforming operations can now be combined. For a beam steered toward  $(\phi, \theta)$ , the combined matched filter is

$$\sum_{x=1 \text{ to } X} w_x(t) a_x(\phi, \theta)$$

This matched filter is applied to the receive beam corresponding to direction  $(\phi, \theta)$ . Doing this for all receive beams, the number of arithmetic operations is reduced to

$$B \cdot N_{MF} + 2N \cdot B_{RX}$$

(note that typically  $B = B_{RX}$ ). Moreover, the memory requirement is reduced, since the  $N$  DAR receiver channels are never processed to form the much larger number (i.e.,  $X \cdot N$ ) of filter channels prior to beamforming. In fact, if  $B_{RX} < N$  there is an early reduction in the amount of data!

## 7. BISTATIC RADAR OPERATION

Ship based bistatic radars may be used for volume search and tracking applications. The benefits of using bistatic radar include a silent receiver (thus LPI), aspect diversity (could get favorable target RCS), and ease of implementing a CW mode (large spatial separation and side-lobe canceling for isolation). The drawbacks of a bistatic system are an increase in processing and/or hardware, requirements to know the transmitter location, knowledge of total signal transit time, and, when using pulse chasing algorithms, orientation of the transmitter [6]. The challenge of implementing a ship based bistatic system is proper compensation for the motion of the two (or more) platforms. The subject of this section is to address some of these concerns for different types of bistatic radars.

To help quantify results, two ship motion models are used. A “moderate seas” model is defined as a three-degree sinusoidal roll over a period of ten seconds. A “heavy seas” model is defined as a ten-degree sinusoidal roll over a period of ten seconds.

Three techniques have been used to ensure the transmit energy is intercepted by the receive beams and a fourth technique is introduced here. These techniques are: use of an omni-beam transmit antenna, use of multiple receive beams to cover the transmit area, pulse chasing algorithms, and use of multiple transmit waveforms from each element in the transmitting array (MIMO radar). Each of these techniques could be used with either a digital receive array (receiver behind each element) or an analog receive array (some techniques require multiple analog beamforming networks to form multiple outputs). The use of a digital receive array is discussed here (for a discussion of the analog receive array see [6]).

The necessary parameter requirements are now discussed for each type of bistatic radar. Regardless of the type of transmitter used and method of receive beamforming, three parameters need to be known in order to solve for the target location (i.e., solving the bistatic triangle): the transmitter location, the transmit phase, and the transit time.

The phase synchronization may be accomplished by means of a communications channel between the transmitter and receiver and the use of a phase-locked-loop to control the phase of the receiver. The phase coherency across a multiple pulse CPI is limited by the ships' motion. Assuming an antenna height of 20 m, the worst case spread across Doppler bins caused by ship motion is shown in Figure 15. To keep the Doppler spread to within a single bin, the coherency time for moderate seas is 0.35 seconds and for

heavy seas, .2 seconds. To lengthen the coherency time, the velocity of the antennas must be tracked and an angular dependent Doppler filter may be used.

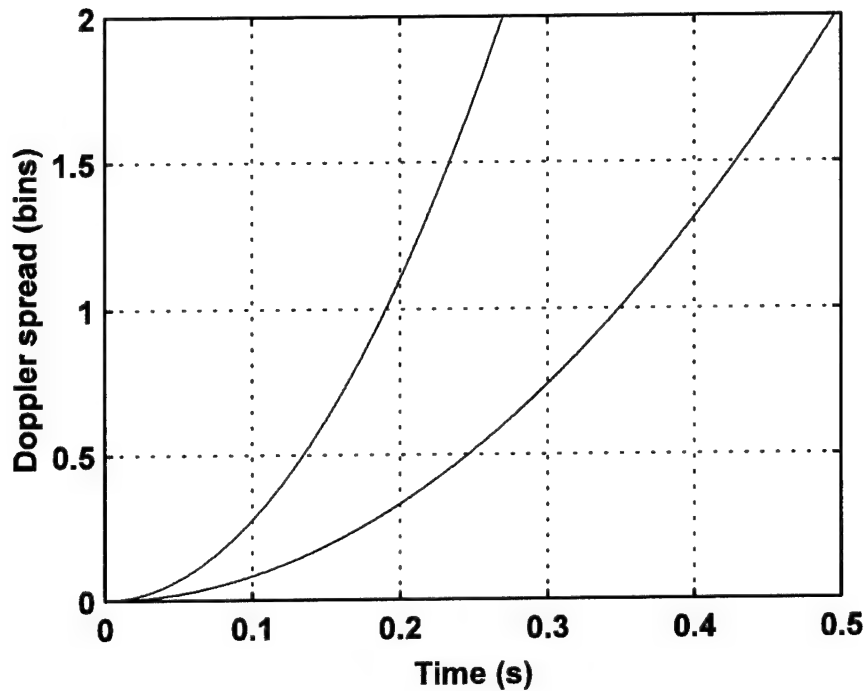


Figure 15. Worst case Doppler spread of signal due to ship motion (blue curve is for moderate seas, red is for heavy seas).

The transmitter location and signal transit time need to be known to estimate the range of the target. Therefore, these parameters should be known to well within the range (time) resolution of the radar (i.e., for a 10 MHz search waveform, the range (time) resolution is 30 m (100 ns)).

For a dedicated omni transmitter, a multi-beam receiver should be used so that the transmitted energy is not wasted. The receiver to use in this case is a multi-beam receiver to cover the transmit beam area. For this type of system, no additional requirements are needed to process the data.



A high gain transmit beam (i.e., narrow beam width) could use several types of receivers. A brute force search of every range cell in every receiver beam, use of pulse chasing [7] to only search the range cells that could have transmit energy, or some combination of the two (e.g., a “rough” pulse chasing algorithm to get close to the range cell of interest and then a small multibeam search around that cell). The brute force method has no additional requirements and works the same as the receiver for the omni transmit. The pulse-chasing algorithm requires knowledge of the transmitter orientation. This new requirement is illustrated in Figure 16. This figure shows two ships in a bistatic configuration. For a particular range cell, the receiver must form the receive beams at the correct angles in order to realize the full antenna gain (illustrated by the black beam positions in the figure). However, if there is an error in the receivers estimate of the transmit array's orientation, the beam will be formed at the wrong angle and thus lead to a loss in signal power (illustrated by the red beam in the figure).

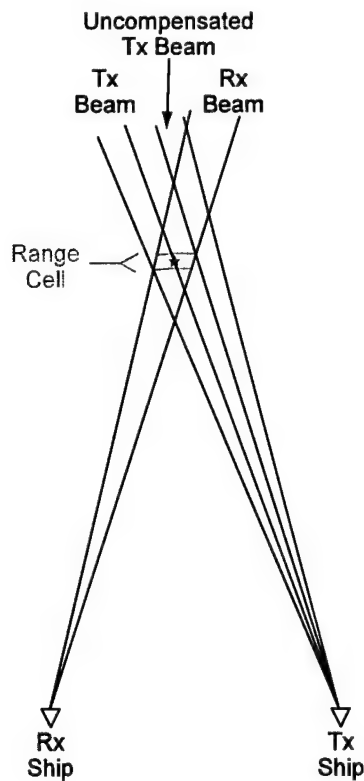


Figure 16. Bistatic radar geometry focusing transmit and receive beams on particular range cell.

To quantify this loss, it is assumed that both the transmit and receive arrays have a 1-degree beamwidth (azimuth and elevation). Figure 17 plots the signal loss due to unknown transmitter orientation as a function of time (zero time corresponds to the receiver correctly updating its position estimate of the transmitter's orientation).

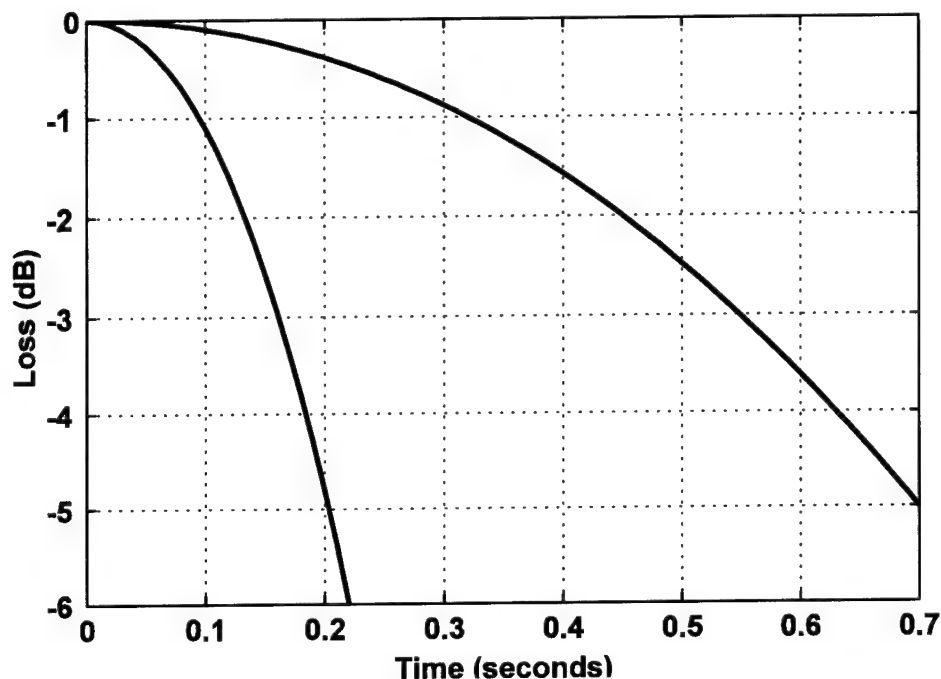


Figure 17. Loss due to uncompensated ship motion assuming 1° beam width.  
Red assumes heavy seas and blue assumes moderate seas.

This implies that, for reasonable losses in signal power (3dB), the receiver must update its estimate of the transmitter's orientation at a rate of 2 to 6 Hz depending on sea conditions. A few possible ways to lessen this requirement would be to track the orientation estimate and predict ahead or estimate the orientation of the transmitter based on a large target that is in track. These loss curves also help in determining the possible length of a CPI. Assuming a stationary target, a CPI's length would be limited to 0.15 to 0.5 seconds unless motion compensation algorithms are used in the beamforming process. A detailed analysis of these issues is left to a later study.

MIMO bistatic radar transmits independent waveforms out of each element (or subarray) for ubiquitous coverage of the search space. The signal processing on the receive side then forms both transmit and receive beams. The processing may be done brute force (search every range cell of every transmit/receive beam combination), use pulse chasing for high efficiency or some combination of the two for robustness. The requirements are the same as for a high gain beam transmit scheme (transmitter orientation required). The benefits of using MIMO bistatic radar are omni coverage without a dedicated omni transmitter, better angular accuracy (angle estimation on both transmit and receive beams), and all other benefits outlined previously in this report.

## 8. OTHER CONSIDERATIONS

There are a number of special topics worth considering with respect to the role played by DAR systems utilizing a MIMO mode. This chapter briefly addresses some of these topics.

### 8.1 ADVANCED MULTIFUNCTION RF SYSTEMS

Advanced multifunction RF systems are a topic of great current interest. As pioneered by ONR, Advanced multifunction RF systems (AMRFs) employ extremely wideband apertures and highly flexible transmitters and receivers. This allows the same set of apertures to be shared among a diverse set of functions, including radar, communications and electronic warfare.

Current efforts to build AMRFs systems have resulted in concepts involving CW (i.e., 100% duty cycle) waveforms. In implementing radar functionality on such an AMRFs system, isolation and dynamic range are important considerations. As such, the MIMO mode DAR concept described in this report (which was shown to improve isolation as well as dynamic range) may therefore be of interest.

The current AMRFs system concept also employs separate transmit and receive apertures. Given a fixed amount of surface area on which to locate these apertures (i.e., on the mast of a Navy ship), the requirement for two apertures (instead of one) can result in several performance penalties. For example, assuming two equal apertures, then each aperture can use no more than half of the available area. The resulting smaller receive aperture will degrade angle estimation performance as compared with a dedicated (single aperture) radar using the same total area. Fortunately, using a MIMO DAR mode will restore some of the lost accuracy, as shown in Chapter 5.

The current AMRFs vision also calls for LPI radar modes. Clearly, the work of Chapter 4 suggests that the MIMO mode DAR concept can help provide this capability.

As for the future, AMRFs system concepts have been proposed that involve passing multiple simultaneous waveforms (e.g., both radar and communications beams) through the same transmitter devices. As noted earlier in this report, this can result in intermodulation distortion, which can degrade radar system stability. Fortunately, the results of Chapter 1 suggest that operating an AMRFs system in a MIMO DAR mode (for purposes of radar search) may mitigate some of these harmful effects.

## 8.2 SUSCEPTIBILITY OF MIMO DAR TO JAMMING

There are two common types of radar jamming employed today, (1) incoherent jamming, and (2) coherent jamming.

Incoherent jammers typically employ noise-like waveforms in an attempt to degrade radar system sensitivity. Assuming the jammer emits a white noise waveform, the  $N \times N$  spatial (receive) jammer plus noise covariance matrix, as measured by an ideal receive-only array, is:

$$\mathbf{R} = \sigma_j^2 \mathbf{b}_j \mathbf{b}_j^H + \mathbf{N}$$

where  $\sigma_j^2$  is the jammer power,  $\mathbf{b}_j$  is the jammer's  $N \times 1$  receive array response vector, and  $\mathbf{N}$  is the covariance of the background noise.

For a MIMO DAR system, the corresponding covariance matrix will be of size  $XN \times XN$  after matched filtering to each of the transmitted waveforms. However, the matched filtering operation does not introduce correlation. Consequently, only receive-array spatial processing can be used to reject interference. That is: MIMO DAR systems will behave like conventional DAR systems with respect to adaptive spatial processing to null (incoherent) jammers.

Coherent jammers (such as DRFMs), in contrast, emit one or more copies (or modified copies) of the radar's own waveform.

In some cases, the coherent jammer is located in the sidelobes of the radar's beam. The jammer attempts to produce an amplified copy of the radar's waveform (possibly delayed) so as to fool the radar into thinking there is a target present at some incorrect angle. In such cases, angle discrimination techniques (typically called "sidelobe blanking") can be used by the radar to discriminate the jammer from a true mainlobe target. The work of Chapter 5 suggests MIMO DAR systems will have superior performance against such jamming.

In other cases, the coherent jammer is located in the mainlobe of the radar's beam. The jammer attempts to produce multiple copies of the radar's waveform at various delays. This creates a sort of "shell game" for most radars, in that the radar must somehow decide which is the true target. In a bistatic configuration, however, MIMO radars can once again use their superior angle resolution to discriminate

between true targets and copies. The idea is illustrated below in Figure 18. In the figure, the target's actual position is along the inner contour of constant bistatic delay. The target re-radiates a copy of the transmitted waveform, with a slight delay. From the bistatic receiver's perspective, this delayed waveform seems to correspond to the outer contour of constant bistatic delay. However, the replica waveform does not come from an angle pair (i.e., transmit angle and receive angle) consistent with any valid target having this delay. If the bistatic MIMO processor forms only beams corresponding to valid targets at this bistatic delay, then the replica will never receive both the full transmit beamformer gain and the full receive beamformer gain. More generally, the nonphysical apparent spatial characteristics of the signal should allow the MIMO receiver to discriminate replicas from true targets.

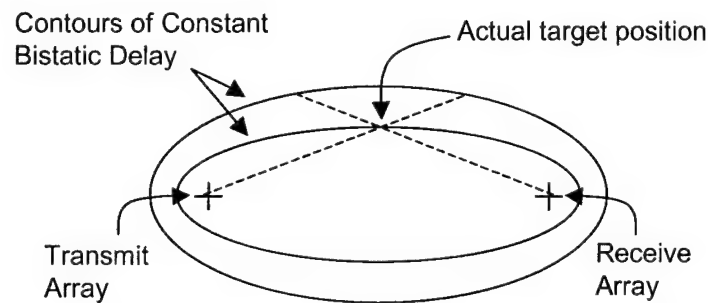


Figure 18. Bistatic MIMO radar subjected to coherent jamming.

### 8.3 ARRAY CALIBRATION

In a MIMO DAR system, each receive sensor's data can be processed by a bank of matched filters (i.e., one filter matched to each transmitted waveform). There will be a total of  $X \cdot N$  matched filter outputs. Each output isolates a single 2-way path (i.e., the path from a single transmit element to a single receive element).

It is thus possible to calibrate both transmit and receive array errors. Similar procedures have been used in satellite communications and in certain radar systems, see [8, 9, 10].

Of course, the ability to better calibrate the transmit array can have a number of benefits (such as better transmit sidelobe control in conventional transmit modes, or the ability to apply low-sidelobe weightings to the “transmit” part of the MIMO beamforming.)

#### **8.4 NCI VS. MIMO PROCESSING**

As mentioned in Chapter 6, one might perform the MIMO signal processing in the following order:

1. Perform matched filtering to each transmit waveform
2. Perform “receive” beamforming
3. Perform “transmit” beamforming

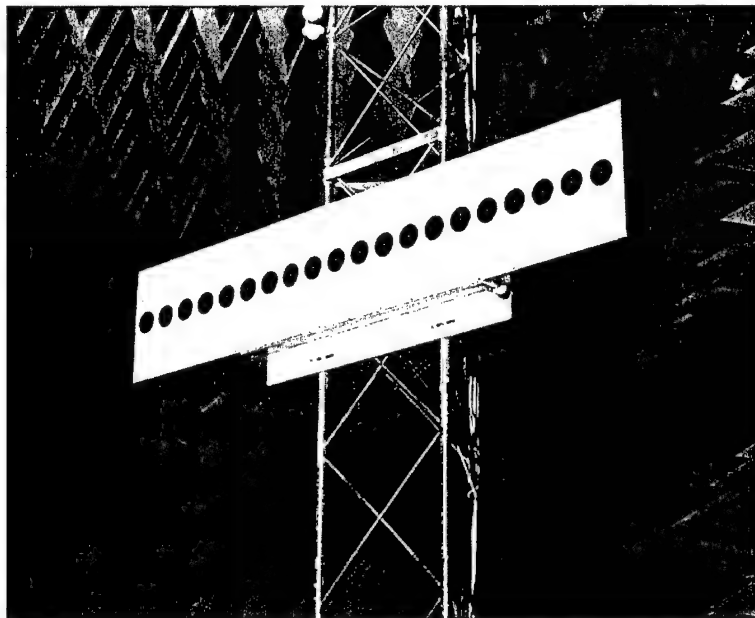
(the order of the later two steps can be reversed). The transmit beamforming operation applies a specific complex weight to each MIMO channel corresponding to each specific transmitter, then sums the transmit channels coherently.

It should be noted that one could instead sum the transmit channels noncoherently (i.e., use noncoherent integration, a.k.a. NCI). Under ideal target and propagation conditions, this would result in a loss in sensitivity. However, under non-ideal conditions (e.g., non planar wavefront propagation) the approach can provide robustness to unknown phase variations introduced along each 2-way signal path. (Note: these perturbations can sometimes be calibrated out adaptively, as in NexGen). Another benefit of using NCI is that one doesn’t need to carefully steer and search over many possible transmit directions.

## 9. PROOF-OF-CONCEPT EXPERIMENTS

In 1999-2001, ONR funded the development of an L-band DAR array at Lincoln Laboratory. The MIMO-based LPI concept described earlier was considered as one mode for operating this array. Due to program funding issues, completion of the L-band DAR testbed was delayed until 2002, when Lincoln Laboratory internally funded the completion of a scaled-back system. The system, as built, contained a re-configurable L-band aperture, with 4 independent transmitters and receivers. The final array hardware is shown below in Figure 19.

Data was collected during late 2002. During one test, waveforms were transmitted and received using the central four elements of the array. A repeater was positioned within the chamber to supply a surrogate test target. The received data was digitally beamformed and MIMO processed to simultaneously form four independent beams. The antenna pedestal was then rotated, allowing the measurement of these four array patterns, as shown in Figure 20. Note that these MIMO-mode array patterns resemble the expected 2-way patterns from a conventional (i.e., single transmit/receive beam) array of the same size. Differences are likely due to calibration issues.



*Figure 19. L-Band testbed array.*



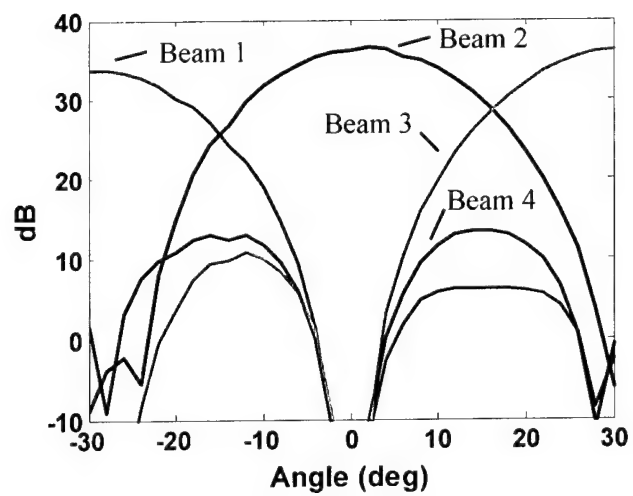


Figure 20. Ubiquitous beams from a 4-element array (measured).

## 10. CONCLUSIONS

This report has proposed ways to perform and use time-energy management in digital array radars. Multifunction DARs can be operated in both conventional and time-energy managed modes. The conventional (i.e., focused transmit beam) modes are well-suited to radar functions requiring high bandwidths and/or short dwells (e.g., precision tracking.) The time-energy managed modes, such as the MIMO mode, are well-suited to functions involving broad searches.

Numerous hardware and system benefits resulting from MIMO operation (and other time-energy managed modes) were described. Hardware benefits include reduced requirements on receiver dynamic range, system stability (phase noise), isolation and spurs. System benefits include improvements in angle accuracy, Doppler resolution, and the ability to operate bistatically and/or in an LPI fashion.

One main conclusion of the report is that fully digital arrays (i.e., arrays that are digital on both transmit and receive) allow a great deal of flexibility in how their transmit energy is spatially distributed and then subsequently collected on receive. The traditional view of digital arrays has often been much more one-sided – focusing mainly on the receive benefits. Digitizing the receive aperture is known to improve system dynamic range, and to support faster search rates (through multiple simultaneous receive beams). Further distributing the system's LO's can potentially provide complementary reductions in phase noise (assuming availability of suitable low-cost, high quality LOs). However, while these are certainly important benefits, they are not the full story with regard to fully digital arrays.

Fully digital arrays also provide increased ability to spatially control the transmitted energy. In doing so, digital arrays can provide complementary decreases in dynamic range, phase noise, and other hardware specifications, as well as various system-level benefits. Certainly, many of the techniques for transmit energy control (e.g., beam spoiling and machine gunning) can be achieved without transmit aperture digitization. However, such approaches do not always scale well to large amounts of transmit energy spreading. Digital arrays can improve such techniques (e.g., by eliminating the time required to reset analog phase shifters between subpulses in a machine-gunned system). Furthermore, digital arrays enable the use of other techniques such as MIMO – which easily scale to very large amounts of transmit energy spreading.

Finally, it is noted that while the analysis in this report has focused mainly on Navy surface-based radar in distributed littoral clutter scenarios, many results are applicable to airborne radars and other types of clutter.

## GLOSSARY

ABF	Analog Beamforming
A/D	Analog-to-Digital Converter
ADC	Analog-to-Digital Converter (alt. form)
DAR	Digital Array Radar
DBF	Digital Beamforming
DDS	Direct Digital Synthesizer
DDC	Direct Digital Downconversion
DRFM	Digital RF Modulation
ELINT	Electronic Intercept
EMI	Electromagnetic Interference
HPA	High Power Amplifier
LNA	Low Noise Amplifier
LO	Local Oscillator
MIMO	Multi-Input, Multi-Output
Rx	Receive
T/R	Transmit and Receive
Tx	Transmit
TAMD	Theater Air and Missile Defense

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13. ABSTRACT (Maximum 200 words) <p>Future navy surface radars will need large Power-Aperture-Gain (PAG) products so as to perform challenging Air and Missile Defense functions. Oftentimes, these radars will operate in littoral regions, where their large PAG products will cause strong clutter returns. Unfortunately, radar equipment specifications can become stressed by the need to detect small targets in such strong clutter. Stressing hardware specifications include dynamic range, phase noise, system stability, isolation, and spurs. Moreover, the additional desire for Low Probability of Intercept (LPI) radar operation will also influence radar hardware design. Hence, as radar PAG increases, it may become increasingly difficult to design conventional radar equipment to operate as desired in littoral regions.</p> <p>This report describes alternative approaches to operating phased array radars, especially digital arrays. These approaches involve transmit-array time-energy management; together, these alternative approaches are shown to ease the stressing hardware requirements described above. Time-energy managed digital arrays, for example, can be used to generate both highly focused transmit beams (e.g., for track) and broad transmit illumination (e.g., for search). Broad transmit illumination provides broad angular coverage analogous to so-called "ubiquitous" radars (i.e., radars that "look everywhere, all the time"). This report describes how such broad transmit illumination can be provided by treating the transmitter and receiver subsystems of the radar as a single MIMO (Multi-Input, Multi-Output) system, or by employing more traditional approaches (e.g., beam spoiling or machine-gunning); pros and cons of each approach are discussed.</p>					
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